Preface

This abstract booklet includes the abstracts of the papers that have been presented at International Conference on Mathematics and its Applications in Science and Engineering (ICMASE 2020) which is held in Ankara Hacı Bayram Veli University, Ankara, Turkey between 9-10 July, 2020, via Online because of Covid 19 pandemia. I hope this pandemic, which covers the whole world, will lose its effect as soon as possible and we will go back to the days before the pandemic.

The aim of this conference is to exchange ideas, discuss developments in mathematics, develop collaborations and interact with professionals and researchers from all over the world in with some of the following interesting topics: Functional Analysis, Approximation Theory, Real Analysis, Complex Analysis, Harmonic and non-Harmonic Analysis, Applied Analysis, Numerical Analysis, Geometry, Topology and Algebra, Modern Methods in Summability and Approximation, Operator Theory, Fixed Point Theory and Applications, Sequence Spaces and Matrix Transformation, Modern Methods in Summability and Approximation, Spectral Theory and Differential Operators, Boundary Value Problems, Ordinary and Partial Differential Equations, Discontinuous Differential Equations, Convex Analysis and its Applications, Optimization and its Application, Mathematics Education, Application on Variable Exponent Lebesgue Spaces, Applications on Differential Equations and Partial Differential Equations, Fourier Analysis, Wavelet and Harmonic Analysis Methods in Function Spaces, Applications on Computer Engineering, Flow Dynamics. However, the talks are not restricted to these subjects only. I am pleased to tell that this conference is also organized as a final multiplier event of the Rules_Math Project, supported by the EU.

Many thanks to all committee members.

We wish everyone a fruitful conference and pleasant memories from ICMASE 2020.

Fatih YILMAZ
Chairman ICMASE 2020
International Conference on Mathematics and Its Applications in Science and Engineering (ICMASE 2020)
09-10 July 2020, Ankara

Honorary and Advisory Board

Yusuf TEKİN, Rector of Ankara Hacı Bayram Veli University, (Turkey)
Gerardo Rodriguez SANCHEZ, Salamanca University, (Spain)

Organizing Committee

Fatih YILMAZ, Ankara Hacı Bayram Veli University, (Turkey) (Conference Chairman)
Deolinda RASTEIRO, Instituto Superior de Engenharia de Coimbra, (Portugal)
Dursun TAŞÇI, Gazi University, (Turkey)
Emel KARACA, Ankara Hacı Bayram Veli University, (Turkey)
Jesús Martín-VAQUERO, Salamanca University, (Spain)
Melek SOFYALIOĞLU, Ankara Hacı Bayram Veli University, (Turkey)
Mücahit AKBIYIK, Beykent University, (Turkey)
Mustafa ÖZKAN, Gazi University, (Turkey)
Selçuk ÖZCAN, Karabük University, (Turkey)

Invited Speakers

Ayman BADAWI, American University of Sharjah, (UAE)
Araceli QUEIRUGA DIOS, Salamanca University, (Spain)
Carlos Martins da FONSECA, Kuwait College of Science and Technology, (Kuwait)

Scientific Committee

Agustín Martín MUNOZ, Spanish National Research Council, (Spain)
Ali Reza ASHRAFI, University of Kashan, (Iran)
Ascensión Hernández ENCINAS, University of Salamanca, (Spain)
Aynur Keskin KAYMAKÇI, Selçuk University, (Turkey)
Carlos Martins da FONSECA, Kuwait College of Science and Technology, (Kuwait)

Cristina R. M. CARIDADE, Instituto Superior de Engenharia de Coimbra, (Portugal)

Cüneyt ÇEVİK, Gazi University, (Turkey)

Daniela RICHTARIKOVA, Slovak University of Technology in Bratislava, (Slovakia)

Daniela VELICHOVA, Slovak University of Technology, (Slovakia)

Engin ÖZKAN, Erzincan Binali Yıldırım University, (Turkey)

Gheorghe MOROSANU, Babes-Bolyai University, (Romania)

Ion MIERLUS-MAZILU, Technical University of Civil Engineering Bucharest, (Romania)

Ji-Teng JIA, Xidian University, (China)

Kadir KANAT, Ankara Hacı Bayram Veli University, (Turkey)

Luis Hernández ENCINAS, Spanish National Research Council, (Spain)

Maria Jesús Santos SANCHEZ, Salamaca University, (Spain)

Marie DEMLOVA, Czech Technical University in Prague, (Czech Republic)

Michael CARR, Technological University Dublin, (Ireland)

Moawwad E.A. EL-MIKKAWY, Mansura University, (Egypt)

Mohammad Sal MOSLEHIAN, Ferdowsi University of Mashad, (Iran)

Murat BEKAR, Gazi University, (Turkey)

Mustafa ÇALIŞKAN, Gazi University, (Turkey)

Nenad P. CAKIC, University of Belgrade, (Serbia)

Nihat AKGÜNÈŞ, Necmettin Erbakan University, (Turkey)

Seda YAMAÇ AKBIYIK, Gelişim University, (Turkey)

Snezhana GOCHEVA-ILIEVA, University of Plovdiv Paisii Hilendarski, (Bulgaria)

Tomohiro SOGABE, Nagoya University, (Japan)

Victor Gayoso MARTINEZ, Spanish National Research Council, (Spain)

Vildan ÖZTÜRK, Ankara Hacı Bayram Veli University, (Turkey)
Contents

The Moore-Penrose Inverse of Symmetric Matrices with Nontrivial Equitable Partitions 1

ABDULLAH ALAZEMI, MILICA ANĐELIĆ, DRAGANA CVETKOVIĆ-ILIĆ

Entrepreneurial Framework Conditions: a PCA Approach in European Countries 2

ALDINA CORREIRA, ANA BORGES, ELIANA COSTA E SILVA, FÁBIO DUARTE

Longitudinal Analysis of Entrepreneurial Framework Conditions on Entrepreneurship Intent 5

ANA BORGES, ALDINA CORREIRA, ELIANA COSTA E SILVA, FÁBIO DUARTE

Training Open Courses to Assess Mathematical Competencies 7

ARACELI QUEIRUGA-DIOS, FATIH YILMAZ, DEOLINDA M.L.D. RASTEIRO, JESÚS MARTÍN VAQUERO

On Computing an Arbitrary Singular Value of a Tensor Sum 9

ASUKA OHASHI, TOMOHIRO SOGABE

On Gdc-Flat Modules 11

AYŞE ÇOBANKAYA, İSMAIL SAĞLAM

A New Approach for Computing the Mock-Chebyshev Nodes 13

B. ALİ İBRAHIMOĞLU

Application of Wendland’s Compactly Supported Functions for the Numerical Simulation of the Space Fractional NLS Equation 14

BAHAR KARAMAN, YILMAZ DERELİ

Cyclic Codes over the Ring $\mathbb{Z}_8 + u\mathbb{Z}_8 + v\mathbb{Z}_8$ 16

BASRİ ÇALIŞKAN

On the Sedenions with $Q$–Integer Components 17

CAN KIZILATEŞ, SELİHan KIRLAK

Assessment with Mathematics Competencies-Based RULES_MATH’s Guides for Calculus I 19

CRISTINA M.R. CARIDADE, DEOLINDA M. L. D. RASTEİRO
Assessment with Mathematics Competencies-Based RULES_MATH’s Guides for Linear Algebra

CRISTINA M.R. CARIDADE, DEOLINDA M. L. D. RASTEIRO

A Study on Constant Angle Ruled Surfaces Generated by Timelike Curve in Minkowski Space

CUMALI EKİÇİ, YASIN ÜNLÜTÜRK, GÜL ÜĞÜR KAYMANLI

Mathematical Competency Assessment-RULES_MATH Guides on Complex Numbers

DANIELA RICHTARIKOVA

Algebraic Structure of $n$-Dimensional Quaternionic Space

DENIZ ALTUN, SALIM YÜCE

Probability and Statistical Methods: Assessing Knowledge and Competencies – Case Study at ISEC

DEOLINDA M. L. D. RASTEIRO, CRISTINA M.R. CARIDADE

Assessing Knowledge and Competencies: RULES_MATH Project’s effects at ISEC

DEOLINDA M. L. D. RASTEIRO, CRISTINA M.R. CARIDADE

A Novel Fractional-Order Model for COVID-19 Infectious with Incommensurate Orders

DIN PRATHUMWAN, KAMONCHAT TRACHOO, INTHIRA CHAIYA

Vehicle and Driver Scheduling Problem using $\varepsilon$-Constraint in Public Transportation

DURDU HAKAN UTKU, BURCU ALTUNOĞLU

New Hyperbolic-Number Forms of the Euler Savary Equation: The Consideration of Future Pointing Timelike Pole Rays for Spacelike Pole Curves

DUYGU ÇAĞLAR, NURTEN GÜRSES

Finite Difference Method for Fractional Order Pseudo-Parabolic Partial Differential Equation

ECEM GÖKTEPE, MAHMUT MODANLI

The Best Known Iteration Bound of Interior Point Methods for Linear Optimization Problem

EL AMIR DJEFFAL, BACHIR BOUNIBANE
A Cluster Analysis on the European Entrepreneurial Framework Conditions over the Last Two Decades

ELIANA COSTA E SILVA, ALDINA CORREIRA, ANA BORGES, FÁBIO DUARTE

Some Characterizations for Split Quaternions

EMEL KARACA, FATIH YILMAZ, MUSTAFA ÇALIŞKAN

A New Approach for Natural Lift Curves and Tangent Bundle of Unit 2-Sphere

EMEL KARACA, MUSTAFA ÇALIŞKAN

Comparison the Solutions between Vector and Set Approaches for Set-Valued Optimization Problems

EMRAH KARAMAN

Generalized Interval-Valued Optimization Problems

EMRAH KARAMAN

On New Narayana Polynomials

ENGİN ÖZKAN, BAHAR KULOĞLU

On $k$-Narayana Sequence

ENGİN ÖZKAN, BAHAR KULOĞLU

Suborbital Graphs of $\hat{\Gamma}_{0,\pi}(N)$ in $\hat{Q}(N)$

ERDAL ÜNLÜYOL, AZIZ BÜYÜKKARAGÖZ

A Bi-Level Mathematical Model to Analyze the Risk at a Hazardous Material Transportation Network

FATIH KASIMOĞLU, ÖZKAN BALİ

The Intrinsic Metric on the Scale Irregular Sierpinski Gasket $SG(2, 3)$

FATMA DIĞDEM KOPARAL, YUNUS ÖZDEMİR

Non-Rotating Frames and its Applications

FATMA KARAKUŞ

Fermi-Walker Parallelism and its Applications
FATMA KARAKUŞ

Dynamics of a Modified a Discrete-Time Predator-Prey Model with Allee Effect 69

FİGEN KANGALGIL, SEVAL İŞİK

Numerical Methods for a Nonlocal Peridynamic Model 71

GIUSEPPE MARIA COCLITE, ALESSANDRO FANIZZI, LUCIANO LOPEZ, FRANCESCO MADDALENA, SABRINA FRANCESCA PELLEGRINO

Double-Diffusion Instability: A Fingering Convection Model for Oceanic Staircase Formation 72

GIUSEPPE MARIA COCLITE, FRANCESCO PAPARELLA, SABRINA FRANCESCA PELLEGRINO

Eigenvalues of the Selfadjoint Schrödinger Operator on Non-compact Star Graph 73

GÖKHAN MUTLU

Evolute Offset of Non-Cylindrical Ruled Surfaces with B-Darboux Frame 75

GÜL ÜGUR KAYMANLI

The Restrictive Sets and their Applications to Soft Group Theory 77

HAKAN AYKUT, AKIN OSMAN ATAGÜN

Mathematical Model of Extinction of *Rastrelliger brachysoma*, Harvesting and Control 79

INTHIRA CHAIYA, KAMONCHAT TRACHOO, DIN PRATHUMWAN

Some Applications of Eigenvalue and Eigenvectors in Engineering 80

ION MIERLUS MAZILU, FATIH YILMAZ

Generalized Quasi-Einstein Normal Metric Contact Pair Manifolds 81

İNAN ÜNAL

A New Approach to Generalized Cantor Set in Fractal Geometry 84

İPEK EBRU KARACAŞAY, SALIM YÜCE

Nonlinear Approximation in Discrete Operators of Sampling-type 86
<table>
<thead>
<tr>
<th>Authors</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>İSMAIL ASLAN</td>
<td>The Way of Assessment of Mathematical Competencies in RULES_MATH Project</td>
<td>88</td>
</tr>
<tr>
<td>JANA GABKOVÁ, PETER LETAVAŬ</td>
<td>Nonlinear Analysis of Heart Rate Dynamics to Estimate the Risk of Cardiovascular Events in Hypertensive Patients</td>
<td>90</td>
</tr>
<tr>
<td>JOSÉ MARÍA LOPEZ-BELINCHON, MIGUEL ÁNGEL LOPEZ GUERRERO, RAÚL ALCARAZ MARTINEZ</td>
<td>Natural Transform Adomian Decomposition Method (Ntadm) for Evaluation of Two Dimensional Schrödinger Black Scholes Time Fractional Ordered Pde, an Application to Financial Modelling</td>
<td>92</td>
</tr>
<tr>
<td>KAMRAN ZAKARIAR, SAEED HAFEEZ</td>
<td>The Study of Axial Load in the Lumbar Spine of Patients with Lumbar Spinal Stenosis by using Finite Element Method</td>
<td>94</td>
</tr>
<tr>
<td>KAMONCHAT TRACHOO, INTHIRA CHAIYA, DIN PRATHUMWAN</td>
<td>Improving Engineering Thermodynamics Learning with Mathematica</td>
<td>95</td>
</tr>
<tr>
<td>MARÍA JESÚS SANTOS, ALEJANDRO MEDINA, JOSÉ MIGUEL MATEOS ROCO, ARACELI QUEIRUGA-DIOS</td>
<td>An Application of Double Sumudu Transform for Solving Telegraph Equation</td>
<td>96</td>
</tr>
<tr>
<td>MARIA AYDIN, HALDUN ALPASLAN PEKER</td>
<td>Triple Sumudu Transform and its Application for Solving Volterra Integro-Partial Differential Equation</td>
<td>98</td>
</tr>
<tr>
<td>MARIA AYDIN, HALDUN ALPASLAN PEKER</td>
<td>Understanding the Failure in Differential and Integral Calculus in the Degrees of Engineering at a Higher Education School in Portugal</td>
<td>100</td>
</tr>
<tr>
<td>MARIA EMÍLIA BIGOTTE DE ALMEIDA, ARACELI QUEIRUGA-DIOS, MARÍA JOSÉ CÁCERES</td>
<td>Analytical Investigation of a Two-Mass System Connected with Linear and Nonlinear Stiffnesses</td>
<td>102</td>
</tr>
<tr>
<td>MD. ALAL HOSEN</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
On Approximation Properties of Szász-Mirakyan Operators

MELEK SOFYALIOĞLU, KADIR KANAT

Cardinalities and Ranks of Nilpotent Subsemigroups of $C_n$

MELEK YAĞCI

The Number of $m$-Potent Elements in $C_{n,Y}$

MELEK YAĞCI

Modelling Tap Water Consumer Ratio

MELTEM EKİZ, OSMAN UFUK EKİZ

Comparison of Nontraditional Optimization Techniques in Optimization of Shell and Tube Heat Exchanger

MERT AKIN INSEL, İNCI ALBAYRAK, HALE GONCE KOCKEN

Fuzzy Modelling on Control of Heat Exchangers

MERT AKIN INSEL, İNCI ALBAYRAK, HALE GONCE KOCKEN

On the New Families of Gauss k-Lucas Numbers and their Polynomials

MERVE TAŞTAN, ENGİN ÖZKAN

On Spaces Derivable from a Solid Sequence Space and a Non-Negative Lower Triangular Matrix

MERVE ULUDAĞ

Numerical Analysis of an Adaptive Non-linear Filter Based Time Regularization Model for the Incompressible Non-Isothermal Fluid Flows

MİNE AKBAŞ, BUSE İNGENÇ

Approximate and Analytical Solutions for Nonlinear Fractional Systems

MUAMMER AYATA, OZAN ÖZKAN

Runge-Kutta-Nystrom Method Based On Arithmetic Mean for Solving Special Second Order Ordinary Differential Equations

MUKADDES ÖKTKEN TURACI
Refinements of Bullen-Type Inequalities for $s$–Convex Functions via Riemann-Liouville Fractional Integrals

MUSA ÇAKMAK

A Generalization of Hermite-Hadamard, Bullen, and Simpson Inequalities via $h$–Convexity

MUSA ÇAKMAK

The Agreement of the Conventional Defect Correction Method and the Novel Defect Correcting Extrapolation Technique for Linear Equations

MUSTAFA AGGUL

On Helicoidal Hypersurfaces in Euclidean 4-Space

MUSTAFA ALTIN, AHMET KAZAN

Periodic Point Results on Quasi Metric Spaces

MUSTAFA ASLANTAŞ

Some Fixed Point Results for Multivalued Mappings on M-Metric Spaces

MUSTAFA ASLANTAŞ, ÜGÜR SADULLAH

k-Order Fibonacci Quaternions

MUSTAFA AŞCI, SÜLEYMAN AYDINYÜZ

Rigid Body Motion of an Elastic Rectangle under the Effect of Sliding Boundary Condition

ONUR ŞAHİN

Stability Analysis and Topological Classification of Fixed Points of Discrete-Time Prey-Predator Model

ÖZGÜR DEMİR, FİGEN KANGALGİL

The Roads We Choose: Critical Thinking Experiences in Mathematical Courses

PAULA CATARINO, MARIA M. NASCIMENTO, EVA MORAIS, PAULO VASCO

A Study on the Tangent Bundle of Riemannian Manifold Endowed with the Bronze Structure

RABIA ÇAKAN AKPINAR
Estimation of Turkey and World Population in the Year 2050 in view of Local and Nonlocal Fractional Derivatives 143

RAMAZAN OZARSLAN

The Effect of Caputo Fractional Derivative Concerning Another Function on Deflection of Horizontal Beams 145

RAMAZAN OZARSLAN

On Lightlike Submersions 147

RAMAZAN SARI

MMR Encryption Algorithm as an Alternative Encryption Algorithm to RSA Encryption Algorithm 148

REMSI AKTAY

Results on Set-valued Prešić Type Mappings 150

SEHER SULTAN YEȘİLKAYA, CAFER AYDIN

New Results on Expansive Mappings 152

SEHER SULTAN YEȘİLKAYA, CAFER AYDIN

On a Type-2 Fuzzy Approach to Solution of Second Order Initial Value Problem 154

SELAMI BAYEĞİ, RAZİYE MERT, TAHİR KHANIYEV, ÖMER AKIN

Existence and Uniqueness Results for a Nonlinear Fractional Differential Equations of Order $\sigma \in (1, 2)$ 156

SINAN SERKAN BİLGİCİ, MÜFIT ŞAN

Assessing Students’ Knowledge and Competences in Mathematics by Combined Tests Using Machine Learning Methods 158

SNEZHANA GOCHEVA-ILIEVA, HRISTINA KULINA, ATANAS IVANOVA

Generalized k-Order Fibonacci Hybrid and Lucas Hybrid Numbers 160

SÜLEYMAN AYDINYÜZ, MUSTAFA AŞCI

k-Order Gaussian Fibonacci Polynomials and Applications to the Coding/Decoding Theory 163

SÜLEYMAN AYDINYÜZ, MUSTAFA AŞCI
Finite Difference Method for Fractional Partial Differential Equation Defined by Atangana-Baleanu Caputo (ABC)  
SÜMEYYE EKER, MAHMUT MODANLI  
On Vertex and Link Residual Domination Numbers of Generalized Caterpillar Graphs  
TUFAŞ TURACI  
Some Constant Angle Spacelike Surfaces Construct on a Curve in De-Sitter 3-Space  
TUĞBA MERT, MEHMET ATÇEKEN  
Hamilton Matrices of Real Quaternions and Spinors  
TÜLAY ERIŞİR, EMRAH YILDIRIM  
Using Freeware Mathematical Software in Calculus Classes  
VİCTOR GAYOSO MARTİNEZ, LUIS HERNÁNDEZ ENCINAS, AGUSTÍN MARTÍN MUÑOZ, ARACELI QUEIRUGA DIOS  
Characterizations and Integral Formula for Generalized Ricci Almost Solitons  
YASEMIN SOYLU  
On Reduction of a (2+1)-Dimensional Nonlinear Schrödinger Equation via Conservation Laws  
YEŞİM SAĞLAM ÖZKAN  
Rings whose g-Semiartinian Modules Have Maximal or Minimal Subprojectivity Domain  
YİLMAZ DURĞUN, AYŞE ÇOBANKAYA  
On the *-Congruence Sylvester Equation  
YUKI SATAKE, TOMOHIRO SOGABE, TOMOYA KEMMOCHI, SHAO-LIANG ZHANG
THE MOORE-PENROSE INVERSE OF SYMMETRIC MATRICES WITH NONTRIVIAL EQUITABLE PARTITIONS

Abdullah ALAZEMI¹, Milica ANDELIĆ², Dragana CVETKOVIĆ-ILIĆ³

¹,²Kuwait University, Kuwait
³University of Niš, Serbia

Corresponding Author’s E-mail: milica.andelic@ku.edu.kw

ABSTRACT

We consider symmetric matrices that admit nontrivial equitable partitions. We determine some sufficient conditions for the quotient matrix of the Moore-Penrose inverse of the initial matrix to be equal to the Moore-Penrose inverse of its quotient matrix. We also study several particular cases when the computation of the Moore-Penrose inverse can be reduced significantly by establishing the formula for its computation based on the Moore-Penrose inverse of the quotient matrix. Among others we consider the adjacency matrix of a generalized weighted threshold graph.

Keywords Equitable partitions · Moore-Penrose inverse · Stepwise matrices

References

Entrepreneurial Framework Conditions: a PCA Approach in European Countries

Aldina CORREIRA¹, Ana BORGES¹, Eliana COSTA E SILVA¹, Fábio DUARTE¹

¹ESTG - School of Management and Technology
²P.PORTO - Polytechnic of Porto
³CIICESI – Center for Research and Innovation in Business Sciences and Information Systems

Corresponding Author’s E-mail: aic@estg.ipp.pt

ABSTRACT

Entrepreneurial Framework Conditions (EFC), as Valliere (2010) [1] defined, are the environmental conditions that encourage and support entrepreneurial activity at the national level. Thereafter Valliere’s work, and until today as illustrate Orobia et. al. (2020) [8], several authors focused on the study of countries EFC. In this work we consider the EFC indicators from Global Entrepreneurship Monitor (GEM), similar to Silva et. al. (2018) [9] and Sampaio et. al. (2016) [7]. The study of Silva et. al. (2018) focus on the twelve indicators of the entrepreneurial ecosystem, defined by the GEM project as EFC and analysed how experts’ perceptions have changed in Portugal between 2010 and 2016, by establishing a comparative analysis between the Portuguese reality and the reality in other European countries. GEM project began in 1999 and is conceived to analyse the phenomenon of entrepreneurship conducting two types of surveys: the Adult Population Survey (APS) and the National Expert Survey (NES).

Correia et. al. (2017) considered NES data-sets for 2011, 2012 and 2013 to study the effects of different type of entrepreneurship expert specialization on the perceptions about the EFC. These data-sets are also considered by Braga et. al. (2018) [4], Queiros et. al. (2019) [5], [10] and Correia et. al. (2016) [3].

According to GEM 2019–20 Global Report, fifty economies participated in the GEM 2019 APS, including: 11 from the Middle East and Africa, 8 from Asia and Pacific, 8 from Latin America and Caribbean, 23 from Europe and North America. 5 of these economies are classified as low-income level, 12 as middle-income and the rest 33 as high-income. Over 150 000 individuals participated in extended interviews as part of the GEM research in 2019.

Thus GEM is a large scale database for internationally comparative entrepreneurship that includes information about many aspects of entrepreneurship activities
of a large number of countries. Because of that is largely study by the academic community.

In this work a Principal Component Analisys (PCA), with varimax rotation approach (Factor Analysis), is implemented for EFC in European countries, during two decades. Our goal is to analyse if the correlation structure of the EFC GEM indicators remains the same during this period or if there are changes in this structure.

Keywords Entrepreneurship · PCA · Factor Analysis · Global Entrepreneurship Monitor (GEM) · Entrepreneurial Framework Conditions (EFC)

Acknowledgment
This work has been partially supported by national funds through FCT – Fundação para a Ciência e Tecnologia through project UIDB/04728/2020.

References


LONGITUDINAL ANALYSIS OF ENTREPRENEURIAL FRAMEWORK CONDITIONS ON ENTREPRENEURSHIP INTENT

Ana BORGES¹, Aldina CORREIRA¹, Eliana COSTA E SILVA¹, Fábio DUARTE¹

¹CIICESI, ESTG, Politécnico do Porto
Corresponding Author's E-mail: aib@estg.ipp.pt

ABSTRACT

Entrepreneurship Intentions are defined as the entrepreneurial orientation, vocational aspirations and the desire to own a business by [5]. This study intents to contribute to the understanding of this emerging concept since it is consensually accepted the major role of entrepreneurship for economic development, job creation and innovation ([4], [3]).

Global Entrepreneurship Monitor (GEM) is a large-scale database for internationally comparative entrepreneurship that includes information about many aspects of entrepreneurship activities of a large number of countries. It is based on collecting primary data through an Adult Population Survey (APS) of at least 2000 randomly selected adults (18–64 years of age) in each economy. In addition, national teams collect experts’ opinions about components of the entrepreneurship ecosystem through a National Expert Survey (NES) ([2]). We aim to understand which GEM Framework Conditions have impact on Entrepreneurship Intentions.

The database used in this work contains the GEM the indicators, corresponding to the Entrepreneurial Behaviour and Attitudes – EBA and Entrepreneurial Framework Conditions – EFC. Subsequent to the study of both the state of the entrepreneurial mind-set, motivations, activities and ambition, and the national framework conditions required to allow entrepreneurship to flourish in an economy. In particular, we analyze how twelve EFCs indicators relate do Entrepreneurial intentions, through a longitudinal study of an unbalance dataset on 28 European countries between 2001 and 2019. Entrepreneurship is heterogeneous across countries, according several GEM data analyses (e.g. Braga et al., 2018, Pilar et al., 2019). Hence, it is important to take into account intra-country specificities when conducting this type of analyses, since company performance can variate regarding the economic level of the country. For that, we tested different specification of mixed-effect longitudinal models to understand which EFC have significant impact on countries entrepreneurial intentions, thought time. Additionally, we also control, similarly to the work of [1] for per-capita GDP, population growth, and industry structure,
since country’s wealth has a significant effect on the character of its entrepreneurial activity [6].

**Keywords** Entrepreneurship Intentions · GEM · Longitudinal models

**Acknowledgment**
This work has been partially supported by national funds through FCT – Fundação para a Ciência e Tecnologia through project UIDB/04728/2020.

**References**


TRAINING OPEN COURSES TO ASSESS MATHEMATICAL COMPETENCIES

Araceli QUEIRUGA-DIOS¹, Fatih YILMAZ², Deolinda M.L.D. RASTEIRO³, Jesús Martín VAQUERO¹

¹Universidad de Salamanca, Salamanca, Spain
²Ankara Haci Bayram Veli University, Ankara, Turkey
³Coimbra Institute of Engineering, Coimbra Polytechnique Institute, Coimbra, Portugal

Corresponding Author’s E-mail: queirugadios@usal.es

ABSTRACT

As part of Rules_Math project a group of 9 European institutions have been working in a set of materials and resources to assess mathematical competencies. “New Rules for Assessing Mathematical Competencies” project gave them the opportunity of sharing their university situation in evaluating mathematics and to analyze and discuss about their country’s current situation in secondary and high school for assessing competencies, and finally, they define the standards for assessing mathematical competencies in engineering education [1, 2]. We present here one of the final outputs of this project: the training courses that have been developed and implemented for training teachers in assessing competencies and for students to acquire the competencies and skills to be competent in mathematics.

The use of mathematical software and tools, with different pedagogical approaches make possible to involve students in this competencies-based teaching and learning process. During the project life several papers have been published that present evidences of the work that have been developed by students and trainers: The use of Wolfram Mathematica through practicums for students [3], game-based learning as a new proposal [4], video-lessons [5], concept maps [6], etc.

The knowledge and the acquisition of mathematical competencies by students was measured with some surveys after using the material developed during the project [7, 8, 9] and the results agree on the usefulness of such a material.

Keywords Engineering Education · Competencies · Learning outcomes

References

[1] Araceli Queiruga-Dios, Ascensión Hernández Encinas, Marie Demlova, Deolinda Dias Rasteiro, Gerardo Rodríguez Sánchez, and María Jesús Sánchez


ON COMPUTING AN ARBITRARY SINGULAR VALUE OF A TENSOR SUM

Asuka OHASHI¹, Tomohiro SOGABE²

¹Ritsumeikan University, Shiga, Japan
²Nagoya University, Aichi, Japan

Corresponding Author’s E-mail: a-ohashi@fc.ritsumei.ac.jp

ABSTRACT

We consider computing a desired singular value of a tensor sum: $T := I_n \otimes I_m \otimes A + I_n \otimes B \otimes I_\ell + C \otimes I_m \otimes I_\ell \in \mathbb{R}^{\ell \times m \times \ell m n}$, where $A \in \mathbb{R}^{\ell \times \ell}$, $B \in \mathbb{R}^{m \times m}$, $C \in \mathbb{R}^{n \times n}$, $I_n$ is the $n \times n$ identity matrix, and the symbol “$\otimes$” denotes the Kronecker product. The tensor sum $T$ arises from a finite difference discretization of three-dimensional constant coefficient partial differential equations. The methods to compute the maximum/minimum singular values of $T$ were provided in [3, 4].

For computing a desired eigenvalue $\lambda$ of a symmetric matrix $M \in \mathbb{R}^{n \times n}$, the Lanczos method with the shift-and-invert technique, see, e.g., [1], is widely known. The method solves the shift-and-invert eigenvalue problem: $(M - \tilde{\sigma} I_n)\mathbf{x} = (\lambda - \tilde{\sigma})^{-1} \mathbf{x}$, where $\mathbf{x}$ is the eigenvector of $M$ corresponding to $\lambda$, and $\tilde{\sigma}$ is a shift point which is set to the nearby $\lambda$ ($\tilde{\sigma} \neq \lambda$). Since the eigenvalue problem has the eigenvalue $(\lambda - \tilde{\sigma})^{-1}$ as the maximum eigenvalue, the method is effective for computing the desired eigenvalue $\lambda$ near $\tilde{\sigma}$.

Therefore, we obtain a method of computing a desired singular value of $T$ based on the shift-and-invert Lanczos method. The method solves a shift-and-invert eigenvalue problem: $(T^T T - \tilde{\sigma}^2 I_{\ell mn})^{-1} \mathbf{x} = (\sigma^2 - \tilde{\sigma}^2)^{-1} \mathbf{x}$, where $\sigma$ is the desired singular value of $T$, $\mathbf{x}$ is the corresponding right-singular vector, and $\tilde{\sigma}$ is set to the nearby $\sigma$ ($\tilde{\sigma} \neq \sigma$). This shift-and-invert Lanczos method needs to solve large-scale linear systems with the coefficient matrix $T^T T - \tilde{\sigma}^2 I_{\ell mn}$. Since the direct methods cannot be applied due to the nonzero structure of the coefficient matrix, the preconditioned conjugate gradient (PCG) method, see, e.g., [1], is applied. However, it is difficult in terms of memory requirements to simply implement this shift-and-invert Lanczos method and the PCG method since the size of $T$ grows rapidly by the sizes of $A$, $B$, and $C$.

In this talk, we present the following two techniques: 1) efficient implementations of the shift-and-invert Lanczos method for the eigenvalue problem of $T^T T$ and the PCG method for $T^T T - \tilde{\sigma}^2 I_{\ell mn}$ using three-dimensional arrays (third-order
tensors) and the $n$-mode products, see, e.g., [2]; 2) a preconditioning matrix based on a structure of $T$ for faster convergence of the PCG method. Finally, we show the effectiveness of the proposed method through numerical experiments.

**Keywords** Shift-and-invert Lanczos method · Tensor sum · Singular value

**References**


ON GDC-FLAT MODULES

Ayşe ÇOBANKAYA¹, İsmail SAĞLAM²

¹Department of Mathematics, Cukurova University, Turkey
²Adana Alparslan Türkeş Science and Technology University, Turkey

Corresponding Author’s E-mail: acaylak@cu.edu.tr

ABSTRACT

Let $E : 0 \rightarrow A \rightarrow B \rightarrow C \rightarrow 0$ be a short exact sequence of modules. It is called gd-closed (respectively, s-closed, d-closed) if $\text{Im}(f)$ is gd-closed (respectively, s-closed, d-closed) in $B$. The class of gd-closed sequences is a proper class and it is projectively generated by g-semiartinian modules. An $R$-module $M$ is said to be flat if the functor $M \otimes_R -$ is exact. It is well-known that $M$ is flat if and only if every short exact sequence $0 \rightarrow A \rightarrow B \rightarrow M \rightarrow 0$ of $R$-modules is pure-exact (see, for example, [6, Proposition 3.67]). Further studies had been arose from the relation between flatness and purity. For example, weakly flat modules and neat flat modules have been studied in [7] and [1]. Note that a module is weakly-flat (respectively, neat-flat), if any short exact sequence ending with $M$ is included in $\text{Closed}$ (respectively, $\text{Neat}$). This studies motivate us to study the modules $M$ such that any short exact sequence ending with $M$ is included in $\text{GD\text{-Closed}}$. A module $M$ gd-closed flat (or shortly gdc-flat) if the kernel of any epimorphism $L \rightarrow M$ is gd-closed in the module $L$. An obvious example of gdc-flat modules is a projective modules. Also, nonsingular modules and modules with zero socle are less obvious examples of gdc-flat modules. It is a fact that a simple module is either singular or projective. We call a module $M$ g-semiartinian if every non-zero homomorphic image of $M$ contains a singular simple submodule. The class of g-semiartinian modules is a torsion class of $G$-Dickson torsion theory and this torsion theory is generated by singular simple modules. The class of modules with projective socle is the torsion-free class of the same torsion theory. Modules $M$ such that each short exact sequence ending with $M$ belongs to $\text{GD\text{-Closed}}$ are investigated by us. In this study, we prove that if the torsion submodule $\tau_{gd}(E)$ of an injective module $E$ is projective, then $E$ is gdc-flat; and the converse is true for $C$-rings. Moreover, for $C$-rings, we prove that all injective modules are gdc-flat if and only if the injective hull of each g-semiartinian module is projective if and only if every semiartinian module embeds in a projective module. Finally, we prove that every $\text{GD\text{-Closed}}$ sequence is a Pure sequence if and only if every module with a projective socle is
flat if and only if every gdc-flat module is flat if and only if every finitely generated module with a projective socle is flat.

Acknowledgement: This work was supported by Research Fund of the Cukurova University. Project Number: 12308

Keywords gdc-flat · g-semiartinian · PS-ring

References

A NEW APPROACH FOR COMPUTING THE MOCK-CHEBYSHEV NODES

B. Ali İBRAHİMOĞLU

Yıldız Technical University Department of Mathematical Engineering, Davutpasa Campus, 34210 İstanbul, Turkey

Corresponding Author’s E-mail: bibrahim@yildiz.edu.tr

ABSTRACT

In the univariate polynomial interpolation, it is well known that equidistant points are unreliable due to the Runge phenomenon. Many different numerical methods have been proposed to defeat this phenomenon. The mock-Chebyshev subset interpolation is one of the best methods to overcome Runge’s phenomenon [1]. The algorithm proposed in the recent paper [2] provides a robust method for computing the mock-Chebyshev nodes for a given set of \((n + 1)\) Chebyshev-Lobatto points. This talk discusses a modification of the algorithm with the same small computational cost. Some numerical experiments are given to show the effectiveness of the proposed procedure.

Keywords: Interpolation · Runge phenomenon · Mock-Chebyshev interpolation

References

APPLICATION OF WENLAND’S COMPACTLY SUPPORTED FUNCTIONS FOR THE NUMERICAL SIMULATION OF THE SPACE FRACTIONAL NLS EQUATION

Bahar KARAMAN¹, Yılmaz DERELİ²

¹,²Department of Mathematics, Eskişehir Technical University, Eskişehir 26470, Turkey

Corresponding Author’s E-mail: bahar_korkmaz@eskisehir.edu.tr

Abstract

This research describes an efficient numerical method based on Wendland’s compactly supported functions to obtain the numerical solution of space fractional non-linear Schrödinger (NLS) equation. Here, the Conformable derivative is used for the fractional derivative according to the space of order \( \alpha \) with \( 1 < \alpha < 2 \). The present numerical discussion is based on two following ways: Firstly, the Crank-Nicolson scheme is employed in the mentioned equation to discretize the space fractional NLS equation, and second, a linear difference scheme is implemented to avoid solving nonlinear systems. In this way, we have a linear, suitable calculation scheme. Then, Wendland’s compactly supported functions are established for this scheme. We need to calculate the conformable fractional derivative of these basis functions because the scheme includes space fractional derivative based on Conformable. To our knowledge, for computing the conformable derivative of a function \( f(t) \), the function must be defined for all \( t > 0 \). The basis function can be scaled to have compact support on \([0, \delta]\) by replacing \( r \) with \( \frac{r}{\delta} \) for \( \delta > 0 \). Here \( \delta \) is called a scaling factor. Therefore, the Conformable fractional derivatives of these functions can be computed. The algorithm of the presented scheme is accurate and easy to apply on computers. In the numerical investigation, the stability analysis of the suggested scheme is examined in a similar way to the classic Von-Neumann technique for the governing equation. Two test problems are used to confirm the efficiency, capability, and simplicity of the method. In our results of computations of approximation solutions, it is worth observing that the choice of order \( \alpha \) changes both the height and the width of the wave, and it also causes two turning points. This situation is expected due to the use of the fractional Schrödinger equation in physics to adapt the shape of the wave without the switch of the nonlinearity and dispersion effects. When the fractional-order \( \alpha \) is two, all obtained results are in

![QR Code Image](https://example.com/qrcode.png)
accord with the proposed technique for the classical NLS equation. Finally, all obtained numerical experiments are presented in tables and figures.

**Keywords** Fractional NLS equation · Crank-Nicolson method · Wendland’s compactly supported functions · Fractional derivative operator · Von-Neumann stability.

**Acknowledgements**
This work is supported by the Scientific Research Projects of Eskişehir Technical University (No: 19ADP108).

**References**


In this paper, we study the cyclic codes over the ring $R = \mathbb{Z}_8 + u\mathbb{Z}_8 + v\mathbb{Z}_8$ where $u^2 = u$, $v^2 = v$, $uv = vu = 0$. We construct the generator polynomials of cyclic codes over $R$. Also, we introduce a Gray map from $R^n$ to $\mathbb{Z}_8^{3n}$ and show that the Gray image of the cyclic codes of odd length $n$ over $R$ is a quasi-cyclic code of index 3 and length $3n$ over $\mathbb{Z}_8$.

**Keywords** Cyclic codes · Quasi-cyclic code · Gray map

**References**


ON THE SEDENIONS WITH $Q$–INTEGER COMPONENTS

Can KIZILATEŞ ¹, Selihan KIRLAK ²

¹,²Zonguldak Bulent Ecevit University, Department of Mathematics, Zonguldak, 67100, Turkey
Corresponding Author’s E-mail: selihankirlak@gmail.com

ABSTRACT

The set of Sedenions, denoted by $\mathbb{S}$, are 16—dimensional algebra. Sedenions are noncommutative and nonassociative algebra over the set of real numbers, obtained by applying the Cayley–Dickson construction to the octonions. Like octonions, multiplication of sedenions are not neither commutative and associative. Sedenions appear in many areas of science, such as electromagnetic theory, linear gravity and the field of quantum mechanics [1, 2, 3, 4, 5]. A sedenion is defined by

$$p = \sum_{i=0}^{15} q_i e_i,$$

where $q_0, q_1, \ldots, q_{15} \in \mathbb{R}$ and $e_0, e_1, \ldots, e_{15}$ are called unit sedenion such that $e_0$ is the unit element and $e_1, e_2, \ldots, e_{15}$ are imaginaries satisfying, for $i, j, k = 1, 2, \ldots, 15$ the following multiplication rules:

$$e_0 e_i = e_i e_0 = e_i, \quad (e_j)^2 = -e_0, \quad (1)$$
$$e_i e_j = -e_j e_i, \quad i \neq j, \quad (2)$$
$$e_i (e_j e_k) = -(e_i e_j) e_k, \quad i \neq j, \quad e_i e_j \neq \pm e_k. \quad (3)$$

The addition of sedenions is defined as componentwise and for $p_1, p_2 \in \mathbb{S}$, the multiplication is defined as follows:

$$p_1 p_2 = \left( \sum_{i=0}^{15} a_i e_i \right) \left( \sum_{j=0}^{15} b_j e_j \right)$$
$$= \sum_{i,j=0}^{15} a_i b_j (e_i e_j),$$

where $e_i e_j$ satisfies the identities (1), (2) and (3). Several generalizations of the well-known sedenions such as Fibonacci sedenions, Lucas sedenions, $k$–Pell and $k$–Pell-Lucas sedenions, Jacobsthal and Jacobsthal-Lucas sedenions, and so on
have been studied by several researchers. For example, in [6], the authors defined the Fibonacci and Lucas sedenions

\[
\hat{F}_n = \sum_{s=0}^{15} F_{n+s} e_s,
\]

and

\[
\hat{L}_n = \sum_{s=0}^{15} L_{n+s} e_s.
\]

Then they also obtained the generating functions, Binet-Like formulas and some interesting identities related to Fibonacci and Lucas sedenions. In this paper, by the help of the \(q\)-integers, we define a new family of sedenions. Then we obtain some special cases for this type of sedenions studied by many researchers before. We also get a number of results for this type of sedenions included Binet-Like formulas, exponential generating functions, summation formulas, Catalan’s identities, Cassini’s identities and d’Ocagne’s identities.

**Keywords** Sedenion algebra · \(q\)-integer · Horadam number

**References**

ASSESSMENT WITH MATHEMATICS COMPETENCIES-BASED RULES_MATH’S GUIDES FOR CALCULUS I

Cristina M.R. CARIDADE¹, Deolinda M. L. D. RASTEIRO¹

¹ Coimbra Institute of Engineering, Portugal

Corresponding Author’s E-mail: caridade@isec.pt

ABSTRACT

This work follows from the “New Rules for assessing Mathematical Competencies” (RULES_MATH) project which aims to change the educational paradigm and to get a common European teaching and learning system based on mathematical competencies rather than contents [https://rules-math.com/]. For this, RULES_MATH describes and analyses ways to assess mathematics taught to future engineers through competencies. The eight mathematical competencies already identified (thinking mathematically; reasoning mathematically; posing and solving mathematical problems; modelling mathematically; representing mathematical entities; handling mathematical symbols and formalism; communicating in, with, and about mathematics and make use of aids and tools for mathematical activity) are recognized in the assessment made to students. The RULES_MATH project partners’ working groups have developed a set of “Guide for a Problem” in the different areas of mathematics that are intended to provide some examples of proposed forms of assessment and competence-based activities. The materials are available to all project partners to apply to different students from different courses and institutions. One of such materials is AC7, which aims to evaluate students about "Methods of integration".

The study was carried out in a group of 80 students of Calculus 1 of the degree in Electrotechnical Engineering at the Engineering Institute of Coimbra. 59 of the students attended the course unit on daytime schedule and 21 students attended on an after-work time.

The results obtained were not satisfactory since at 5 questions (out of 9) the mean are less than 50% and in one of them all the students have grades less than 50%. Relating to the test itself as a tool to assess competencies, we may conclude that it covers most of the competencies that we need to evaluate and its difficulty is adequate to our students. However, many students need a personal attention like office hours or extra work, because their results were clearly negative.
**Keywords** Assessment · Significant Learning · Competencies · Mathematics · Engineering

**References**


ASSESSMENT WITH MATHEMATICS COMPETENCIES-BASED RULES_MATH’S GUIDES FOR LINEAR ALGEBRA

Cristina M.R. CARIDADE¹, Deolinda M. L. D. RASTEIRO¹

¹ Coimbra Institute of Engineering, Portugal

Corresponding Author’s E-mail: caridade@isec.pt

ABSTRACT

This work follows from the “New Rules for Assessing Mathematical Competencies” (RULES_MATH) project which aims to change the educational paradigm and to get a common European teaching and learning system based on mathematical competencies rather than contents [https://rules-math.com/]. For this, RULES_MATH describes and analyses ways to assess mathematics taught to future engineers through competencies. The eight mathematical competencies already identified (thinking mathematically; reasoning mathematically; posing and solving mathematical problems; modelling mathematically; representing mathematical entities; handling mathematical symbols and formalism; communicating in, with, and about mathematics and make use of aids and tools for mathematical activity) are recognized in the assessment made to students. The RULES_MATH project partners’ working groups have developed a set of “Guide for a Problem” in the different areas of mathematics that are intended to provide some examples of proposed forms of assessment and competence-based activities. The materials are available to all project partners to apply to different students from different courses and institutions. Two of such materials are LA3 and LA4, which aims to evaluate students about "Matrices and determinants" and "Solution of simultaneous linear equations", respectively.

The study was carried out in a group of 143 students of Linear Algebra from the degree in Biomedical Engineering (20 students), Electromechanics (39 students) and Mechanics (84 students) from the Engineering Institute of Coimbra.

The results obtained were satisfactory since at almost questions (9 out of 11) students were able to obtain a positive grade. The performed assessment test also permits to identify level differences between the courses where it was applied. Relating the test itself as a tool to assess competencies, we may conclude that it covers most of the competencies that we need to evaluate, and its difficulty is adequate to our students.
Keywords Assessment · Significant Learning · Competencies · Mathematics · Engineering

References


A STUDY ON CONSTANT ANGLE RULED SURFACES Generated by Timelike Curve in Minkowski Space

Cumali EKİÇİ¹, Yasin ÜNLÜTÜRK², Gül Uğur KAYMANLI³

¹Department of Mathematics and Computer Science, Eskisehir Osmangazi University Eskisehir, 26040, Turkey
²Department of Mathematics, Kırklareli University, Kırklareli, 39100, Turkey
³Department of Mathematics, Cankiri Karatekin University, Cankiri, 18100, Turkey

Corresponding Author’s E-mail: gulugur@karatekin.edu.tr

ABSTRACT

In geometry, a constant angle surface in three dimensional space is a surface whose tangent planes make a constant angle with a fixed vector field of the ambient space and a ruled surface in three dimensional space is a surface which can be described as the set of points swept by a moving straight line. These constant angle ruled surfaces generalize the concept of general helix, that is, curves whose tangent vectors make a constant angle with a fixed vector. General helix is characterized by the fact that the ratio of the torsion and curvatures of the curve is constant. In addition to this, the slant helix is a curve whose normal vectors make a constant angle with a fixed direction and it is characterized by the fact that the geodesic curvature of the principal image of the principal normal indicatrix is a constant function. In this study, we work on constant angle ruled surfaces generated by timelike curve with respect to Frenet frame \{t, n, b\} in three dimensional Minkowski space \(E^3_1\). We also deal with constant angle ruled surfaces parallel to both tangent of general helix and normal of slant helix. That is, we take the tangent vector and normal vector of a ruled surface are linearly dependent to the tangent vector \(t\) and normal vector \(n\) of the timelike base curve \(α(t)\), respectively. We then give some characterizations such as minimality, developability (i.e. flatness) by using both Gaussian and mean curvatures for these surfaces using Frenet frame in three dimensional Minkowski space from the point of view the constant angle property. In this setting we are able to show that the constant angle ruled surfaces generated by both tangent and normal vectors are developable (i.e. these constant angle ruled surfaces have vanishing Gaussian curvature) in three dimensional Minkowski space.

Keywords Constant Angle Surface · Ruled Surface · Timelike Curve
References

MATHEMATICAL COMPETENCY ASSESSMENT – RULES_MATH GUIDES ON COMPLEX NUMBERS

Daniela RICHTARIKOVA

Slovak University of Technology in Bratislava Institute of Mathematics and Physics, Faculty of Mechanical Engineering, Slovakia

Corresponding Author’s E-mail: daniela.richtarikova@stuba.sk

ABSTRACT

Assessment makes an integral part in education. Generally, it is considered to be the representative of a level of mastering some specialisation, and gives a promise for future that the person will be able to use the achieved knowledge and skills in further study, occupation or common life. Assessment comprises several very important roles. It provides the feedback of study process and its results for the person oneself, and for his or her educator as well. It reveals strengths and weaknesses, gives motivation for further study, or decides about admission to following grades of study or to future employment. Speaking about assessment of a student within a course, it gives the information about the level of student’s knowledge and skills on some curricula or a curricula unit. On one hand, it reflects the level of mastering the content, what is in general took into consideration predominantly, and on another hand, it is usually automatically supposed to reflect also the level of corresponding competency. Regard to competency education becomes very important in these days and the international team of Rules_Math Erasmus + project developed the Guides focussed especially for training and assessment of mathematical competency for students of technical tertiary education. With respect to the SEFI MWG Group document “A Framework for Mathematics Curricula in Engineering Education” (Alpers et al., 2013), the Guides were build upon Core 1 contents – mathematics curricula units studied mostly at bachelor technical degree, distinguishing eight main mathematical competencies proposed by the Danish KOM project in two thousands (Niss, 2003).

In the paper, we deal with the Guide on Complex Numbers prepared in the Department of Mathematics and Physics at the Faculty of Mechanical Engineering STU, the partner of the project, and introduce a competencies evaluation methodology. The results of testing, and the teaching methods of best practice for competency development are presented too.

Keywords Mathematical competency · Assessment · Technical university · Complex numbers
References


ALGEBRAIC STRUCTURE OF \( n \)-DIMENSIONAL QUATERNIONIC SPACE

Deniz ALTUN \(^1\), Salim YÜCE \(^2\)

\(^1,2\) Yıldız Technical University, Faculty of Arts and Sciences, Department of Mathematics Istanbul, 34220, Turkey

Corresponding Author’s E-mail: deniz.altun@outlook.com (https://orcid.org/0000-0001-8327-1161), sayuce@yildiz.edu.tr (https://orcid.org/0000-0002-8296-6495)

ABSTRACT

Real quaternions, first described by W. R. Hamilton in 1843, are a 4-dimensional number system \([7, 8, 9]\). Real quaternions have been studied in many areas such as algebra, geometry, physics, computer-aided design (CAD), put into practice and have provided technological improvements.

Just as the elements of an \( n \)-dimensional real numbers space are called vectors, the elements of \( n \)-dimensional real quaternion space can also be called \( n \)-vectors. The \( n \)-dimensional quaternionic space is a \( 4n \)-dimensional vector space over the real numbers. Regarding this space, studies are carried out in areas such as symplectic geometry, Clifford algebra, etc \([4, 2, 1, 6]\).

In the literature, various ordering types are given on the \( n \)-dimensional real numbers space and the component-wise product is defined \([5, 3]\). In this study, first of all, a metric is defined depending on the order relation of the \( n \)-dimensional real numbers set. At the same time, ring structure was established in \( n \)-dimensional quaternionic space by considering the component-wise product which is defined in the \( n \)-dimensional real numbers set.

Following that, the set of \( n \)-dimensional quaternions is also defined by a different representation of \( n \)-vectors. Using this notation, formulations corresponding to the basic operations in \( n \)-dimensional quaternionic space are obtained. In addition, by adhering these representations we mentioned, vector space and module structures of \( n \)-dimensional quaternionic space over the set of real ordered \( n \)-tuples and over the set of \( n \)-dimensional real numbers were obtained.

The definitions, theorems and results we provide in this study are preliminary information in terms of our research in the field of differential geometry in \( n \)-dimensional quaternionic space. In the light of this study, it is aimed to add the theory of curves in the \( n \)-dimensional quaternionic space to the literature.

Keywords Symplectic geometry · \( n \)-dimensional quaternionic space · Vector space
References

PROBABILITY AND STATISTICAL METHODS: ASSESSING KNOWLEDGE AND COMPETENCIES – CASE STUDY AT ISEC

Deolinda M. L. D. RASTEIRO¹, Cristina M.R. CARIDADE¹

¹Coimbra Institute of Engineering, Portugal
Corresponding Author’s E-mail: dml@isec.pt

ABSTRACT

The concepts taught during a Statistical Methods course make use of different mathematical skills and competencies. The idea of presenting a real problem to students and expect them to solve it from beginning to end is, for them, a harder task then just to obtain the value of a probability given a known distribution. Much has been said about teaching mathematics related to day life problems. In fact, we all seem to agree that this is the way for students to get acquainted of the importance of the contents that are taught and how they may be applied in the real world. The definition of mathematical competence as was given by Niss (Niss, 2003) means the ability to understand, judge, do, and use mathematics in a variety of intra– and extra – mathematical contexts and situations in which mathematics plays or could play a role. Necessarily, but certainly not sufficient, prerequisites for mathematical competence are lots of factual knowledge and technical skills, in the same way as vocabulary, orthography, and grammar are necessary but not sufficient prerequisites for literacy. In the OEDC PISA document (OECD, 2009), it can be found other possibility of understanding competency which is: reproduction, i.e, the ability to reproduce activities that were trained before; connections, i.e, to combine known knowledge from different contexts and apply them do different situations; and reflection, i.e, to be able to look at a problem in all sorts of fields and relate it to known theories that will help to solve it. The competencies that were identified in the KOM project (Niss, 2003, Niss & Højgaard, 2011) together with the three “clusters” described in the OECD document referred above were considered and adopted will slightly modifications by the SEFI MWG (European Society for Engineering Education), in the Report of the Mathematics Working Group (Alpers, 2013). At Statistical Methods courses often, students say that assessment questions or exercises performed during classes have a major difficulty that is to understand what is asked meaning the ability to read and comprehend the problem and to translate it into mathematical language. The study presented in this paper reflects an experience performed with second year students of Mechanical Engineering graduation of Engineering...
Institute of Coimbra, where the authors assessed statistical methods contents taught during the first semester of 2017/2018 academic year. The questions assessment tests were separated into two types: ones that referred only to problem comprehension and its translation into what needed to be calculated and others where students need only to apply mathematical techniques in order to obtain the results. This paper is one of the results of RULES_MATH project which aims to develop tools to assess mathematical competencies. Eight mathematical competencies identified are recognized in the assessment made to students in what concerns learning probability theory concepts. Since 2017 a study was carried out with Mechanical Engineering students at Engineering Institute of Coimbra. The results obtained cover the test as a tool to assess competencies and, also its fitness to our students.

**Keywords** Assessment · Significant Learning · Competencies · Mathematics · Engineering

**References**


ASSESSING KNOWLEDGE AND COMPETENCIES: RULES_MATH PROJECT’S EFFECTS AT ISEC

Deolinda M. L. D. RASTEÎRO¹, Cristina M.R. CARIDADE¹

¹Coimbra Institute of Engineering, Portugal
Corresponding Author’s E-mail: dml@isec.pt

ABSTRACT

In higher education, mathematics has an important role in engineering courses. From any engineering course curriculum there are, in the first year, curricular units (CU) in the area of mathematics that are fundamental for students to acquire the necessary basic knowledge to the most specific CU of their course. Without this well-established mathematical foundation, success in the engineering CUs is seriously compromised. During an engineering course, students learn and consolidate the basic principles of mathematics to solve practical problems, reinforcing their mathematical concepts knowledge. However, although mathematics is a basic discipline in admission to engineering courses, difficulties are identified by engineering students in CUs related to mathematic basic core. In this context, it seems pertinent to identify the mathematics competencies attained by engineering students so that they can use these skills in other CU and take them as tool support to their professional activities. In this talk we present some results obtained while participating in project RULES_MATH, namely ways to assess competencies in calculus, algebra and statistical methods curricular units. The study presented in this paper reflects an experience performed with second year students of Mechanical Engineering graduation and with first year students of Electrotechnical Engineering, Biomedical Engineering, Electromechanics and Mechanics of Engineering Institute of Coimbra, where the authors assessed curricular units contents taught during the first semester of 2017/2018 to 2019/2020 academic years. This paper is one of the results of RULES_MATH project which aims to develop tools to assess mathematical competencies. The eight mathematical competencies already identified (thinking mathematically; reasoning mathematically; posing and solving mathematical problems; modelling mathematically; representing mathematical entities; handling mathematical symbols and formalism; communicating in, with, and about mathematics and make use of aids and tools for mathematical activity) are recognized in the assessment made to students in what concerns learning concepts. The authors
will present the changes performed in their curricular units as a result of the project development and show how the reaction of students was.

Keywords Assessment · Significant Learning · Competencies · Mathematics · Engineering

References


A NOVEL FRACTIONAL-ORDER MODEL FOR COVID-19 INFECTIOUS WITH INCOMMENSURATE ORDERS

Din PRATHUMWAN¹, Kamonchat TRACHOO², Inthira CHAIYA²

¹Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand
²Department of Mathematics, Faculty of Science, Mahasarakham University, Mahasarakham 44150, Thailand

Corresponding Author’s E-mail: inthira.c@msu.ac.th

ABSTRACT

It is well known that the spread of the novel coronavirus (COVID-19) outbreak has affected people and economics around the world. This outbreak was first found in Wuhan, China. The number of cases is increasing every day. The understanding of the dynamics of infectious disease becomes an important way to control the pandemic. Mathematical modeling is an essential role to predict and understand the behavior of the disease spreading. One of the most powerful tools for forecasting is a fractional-order model because the fractional-order model provides more realistic than the traditional integer order model. In this research, we propose the fractional-order system of differential equations with incommensurate orders to predict the trend of disease spreading, and the model examines the dynamics of disease transmission for various orders theoretically and numerically.

Keywords  Incommensurate fractional order derivative · COVID-19 · Equilibrium points · Epidemic model

References


VEHICLE AND DRIVER SCHEDULING PROBLEM USING \(\varepsilon\)-CONSTRAINT IN PUBLIC TRANSPORTATION

Durdu Hakan UTKU\(^1\), Burcu ALTUNOĞLU\(^1\)

\(^1\)University of Turkish Aeronautical Association, Bahçekapı Mah. Okul Sk. No:11, 06790 Etimesgut, Ankara, Turkey

Corresponding Author’s E-mail: dhutku@thk.edu.tr

ABSTRACT

In this study, a multiple objective mixed integer programming model for vehicle and driver scheduling problem is applied to a public transportation system. The first objective function has the operational objective and tries to determine the optimal number of vehicles and drivers to be used to provide the minimum cost. The second objective minimizes the transfer times which related with the satisfaction of the customers. The \(\varepsilon\)-constraint method is used for the multi-objective public transportation model. In the \(\varepsilon\)-constraint method, while one of the objectives reaches the optimum value, the other objective function is solved within the bounds of the \(\varepsilon\)-constraint. The function that minimizes the cost in the proposed model is designated as the objective function and the other objective function, which minimizes the waiting time at the transfer centers, is included in the model as a constraint. The costs include the total fixed costs of drivers and vehicles, as well as the variable costs caused by deadhead and d-trips. Each d-trips must be assigned to the appropriate vehicle and driver without any conflict. D-trips belonging to the same route must be assigned to the same vehicle. The other objective function that provides the minimum waiting time is restricted by the \(\varepsilon\)-constraint method. The waiting time is calculated by considering the time the vehicles reach these transfer points and spend at the transfer points. The model considers the specified waiting times as the expected waiting times. In the first phase, the model determines the cost by having to assign a driver to each vehicle after determining the appropriate number of vehicles. Drivers are selected by considering the salary they get. These salaries include fixed and overtime component. In the second phase, the objective function, which provides minimum cost, determines the number of buses to be kept in stock and considers their holding costs. This part, which makes the waiting time minimum in the transfer centers, achieves its objective without paying attention to the cost. The transfer centers at the intersection of the d-trips are areas that many passengers use and sometimes must wait a lot. The waiting time is calculated by considering the time the vehicles reach these transfer points and spend at the transfer points.
The test problems have been solved with GAMS/CPLEX solver to determine the performance of the model.

**Keywords** Vehicle and Driver Scheduling · Multi-Objective Mixed Integer Programming · $\varepsilon$-constraint Method

**References**


NEW HYPERBOLIC-NUMBER FORMS OF THE EULER SAVARY EQUATION: THE CONSIDERATION OF FUTURE POINTING TIMELIKE POLE RAYS FOR SPACELIKE POLE CURVES

Duygu ÇAĞLAR 1, Nurten GÜRSES 1

1 Yıldız Technical University, Faculty of Art and Sciences, Department of Mathematics, 34220, Istanbul, Turkey
Corresponding Author’s E-mail: duygucaglar94@gmail.com (https://orcid.org/0000-0003-2036-9684), nbayrak@yildiz.edu.tr (https://orcid.org/0000-0001-8407-854X)

ABSTRACT

In 2-dimensional Euclidean plane $\mathbb{E}^2$, one-parameter planar motions are introduced by Blaschke and Müller [1]. Also in [1], the relation between velocities and accelerations is examined and the Euler Savary Equation is calculated during the motion. In addition, in the complex plane $\mathbb{C}$, one parameter planar motions are examined in [1] and Euler Savary Equation is obtained by presenting the relation between the curvatures of the trajectory curves in [2]. In analogy with complex motions, one parameter motions in the hyperbolic plane considering hyperbolic numbers (the numbers of the form $z = x + jy$ where $x$ and $y$ are real numbers and $j^2 = 1, j \neq \pm 1$, see in [3]-[11]) is defined by [12]. Also, Euler Savary Equation in the hyperbolic plane is determined by [13].

Euler Savary Equation gives the radius of curvature and the center of curvature of the path traced by a point in moving plane. In [14]-[16], the new complex-number forms of the Euler Savary Equation are presented in a computer-oriented format. By using these forms, if any three of the following four points: "pole point, arbitrary point of moving plane, inflection point and center of curvature of the path traced by the point moving plane" are known, the fourth one can be found. This complex number technique has the advantage of eliminating the need for the traditional sign conventions and is suitable for digital computation, [14]-[16].

By inspired of the complex number approach, and using the basic concepts in [17]-[19], we examine new hyperbolic number forms of the Euler Savary Equations in this study for the following case: We firstly categorize all of the situations in hyperbolic plane and examine the calculations for future pointing timelike pole rays by considering spacelike pole curves in analog with the studies [14]-[16]. These new hyperbolic forms give not only the relationship between a point of moving plane and center of curvature of the path traced by the point moving plane but...
also the relation between the above mentioned four-points. This hyperbolic number method based on vector calculations and provides a detailed examination of the locations of the points in the hyperbolic plane. Finally, an example is presented to illustrate this case.

**Keywords** One-parameter planar motion · The Euler Savary Equation · Hyperbolic Number

**References**


FINITE DIFFERENCE METHOD FOR FRACTIONAL ORDER PSEUDO-PARABOLIC PARTIAL DIFFERENTIAL EQUATION

Ecem GÖKTEPE 1, Mahmut MODANLI 2

1,2 Harran University, Turkey

Corresponding Author’s E-mail: ecem.goktepe@icloud.com

ABSTRACT

In this paper fractional pseudo-parabolic equation with initial-boundary conditions is investigated. Finite difference schemes are constructed for this differential equation. Stability estimates are proved for these difference schemes. Exact solution of this differential equation is calculated by Laplace transform method. Error analysis are made by comparing the exact solution and the approximate solutions. Figures showing the physical properties of the exact and approximate solutions are given. From the error analysis table and figures, it is clearly seen that this applied method is an effective and good method for this equation.

Keywords Finite difference method · Fractional order pseudo-parabolic partial differential equation · Stability

References

THE BEST KNOWN ITERATION BOUND OF INTERIOR POINT METHODS FOR LINEAR OPTIMIZATION PROBLEM

El Amir DJEFFAL¹, Bachir BOUNIBANE²

¹LEDPA Laboratory, University of Batna 2, Batna, Algeria
²University of Batna 2, Batna, Algeria

Corresponding Author’s E-mail: l.djeffal@univ-batna2.dz

ABSTRACT

Kernel functions play an important role in the complexity analysis of the interior point methods for linear optimization. In this paper, we present a primal dual interior point method for linear optimization based on a new parameterized kernel function. The proposed kernel function is a generalization of the one used recently in Bai et al. (SIAM J Optim 15:101128, 2004) for $LO$, and they showed that the iteration bound for the corresponding algorithm is $O(\sqrt{n} \log n)^2 \log \frac{n}{\epsilon}$. By using several new technical lemmas, the iteration complexity bound as $O(\sqrt{n} \log n) \log \frac{n}{\epsilon}$ is obtained, which coincides with the currently best iteration complexity bounds for large-update methods. The numerical tests confirm that the proposed algorithm is robust.

Keywords Kernel function · Linear optimization · Primal-dual interior-point methods · Large-and small update methods

References

A CLUSTER ANALYSIS ON THE EUROPEAN ENTREPRENEURIAL FRAMEWORK CONDITIONS OVER THE LAST TWO DECADES

Eliana COSTA E SILVA¹, Aldina CORREIRA¹, Ana BORGES¹, Fábio DUARTE¹

¹CIICESI, ESTG, Politécnico do Porto
Corresponding Author’s E-mail: eos@estg.ipp.pt

ABSTRACT

Most policymakers and academics agree that entrepreneurship is critical to the development and well-being of the society [1]. In fact, entrepreneurs create jobs, and drive and shape innovation. Therefore, entrepreneurship is a catalyst for economic growth and national competitiveness. The Global Entrepreneurship Monitor (GEM) research project, founded in 1997, is the largest ongoing study of entrepreneurial dynamics in the world [2]. According to GEM 2019/2020 Global Report, fifty economies participated in the GEM 2019 Adult Population Survey (APS), including 21 European countries. The present study focus on the twelve indicators of the entrepreneurial ecosystem, defined by GEM, i.e. the Entrepreneurial Framework Conditions (EFCs). Specifically, the aim is to study the changes that have occurred in the Portuguese experts’ perceptions over the last two decades, while comparing them to the perceptions of experts on other European countries. Many studies have used GEM data for their research [4], [5], [6], [7], [8]. In [3], the period from 2010 to 2016 is analysed. Substantial changes on the clusters of European through these years were observed. In particular, it was found that despite the economic and financial similarities between Portugal, Italy, Greece and Spain, that faced a dramatic period between 2010-2012, Portugal took off from the remaining countries after 2012, and only in 2016 it was caught up by Spain. The present study aims at extending that work by considering the period before the crisis and after 2016, in order to obtain a wider view on the European entrepreneurs’ perceptions. Several multivariate cluster analysis techniques are used to group the European economies according to the experts’ perceptions on the EFCs of their country.

Keywords Cluster Analysis · Entrepreneurial Framework Conditions · Global Entrepreneurship Monitor

Acknowledgment

This work has been partially supported by national funds through FCT – Fundação para a Ciência e Tecnologia through project UIDB/04728/2020.
References


SOME CHARACTERIZATIONS FOR SPLIT QUATERNIONS

Emel KARACA¹, Fatih YILMAZ², Mustafa ÇALIŞKAN³

¹,² Department of Mathematics, Ankara Hacı Bayram Veli University, Ankara, Turkey
³ Department of Mathematics, Gazi University, Ankara, Turkey

Corresponding Author’s E-mail: emel.karaca@hbv.edu.tr

ABSTRACT

Quaternions, introduced by W. Hamilton, plays a significant role in many areas of science; such as differential geometry, theory of relativity, engineering, etc. Although they have been studied by many researchers in recent years, they yield interesting results every time. In this study, some properties of split quaternions with quaternion coefficients are investigated. Moreover, some different properties between split quaternions with quaternion coefficients and quaternions with complex coefficients are compared.

Keywords Quaternions · Split quaternions · Quaternions with quaternion coefficients

References

A NEW APPROACH FOR NATURAL LIFT CURVES AND TANGENT BUNDLE OF UNIT 2 SPHERE

Emel Karaca 1, Mustafa Çalışkan 2

1 Ankara Hacı Bayram Veli University, Department of Mathematics, Ankara, Turkey
2 Gazi University, Department of Mathematics, Ankara, Turkey

Corresponding Author’s E-mail: emel.karaca@hbv.edu.tr

ABSTRACT

In this study, some characterizations of Bertrand, involute-evolute curve couples for natural lift curves were considered. Obtained results were restricted to the subset of tangent bundle of unit 2-sphere. Moreover, conditions for these curves to be Bertrand and involute-evolute curve couples were given. Finally, some examples were represented to support the main results.

Keywords Natural lift curve · tangent bundle · Involute-evolute curve couple · Bertrand curve

References

We encounter the optimization problems (mathematical programming) in our life. When these problems are expressed mathematically, an objective function emerges. Naturally, solution methods of these problems have attracted the attention to researchers working in the field such as mathematics, engineering, computer science, economics, business, etc. for years. Optimization problems are classified according to the objective function. For example, the problem is called a set-valued optimization problem (shortly, set optimization problem) when the objective function is a set-valued map. Set optimization problems are a generalization of the scalar, vector, and interval optimization problems because the set-valued maps are generalizations of the real-valued, vector-valued, and interval-valued functions, respectively. There are some types of solution concepts in the set-valued optimization problems as vector approach and set optimization approach. For further information about these approaches, one can see in [6, 1, 2, 3, 4, 5]. In the vector approach, the solutions that give efficient (minimal or maximal) points of image set of the objective set-valued map are looked at. Set optimization was introduced by Kuroiwa [6]. This approach depends on comparisons among values of the set-valued map and this concept requires order relations to compare sets. Kuroiwa et al. [9] introduced six order relations for sets. Later, Jahn and Ha [8] defined new order relations and gave some properties of them for set optimization problems.

In this study, solutions are compared between vector and set approaches for set-valued optimization problems, where solutions are obtained by lower set less, upper set less, and set fewer order relations for set approach. Because these order relations are pre-order relations, we use the definition of the minimal and maximal elements for given pre-order relations. Also, according to these approaches, the relationships between the solutions of a set optimization problem are shown in the examples. Moreover, it has been shown on the examples that a set optimization problem, which
doesn’t have any solution according to the vector approach, has some solutions according to the set approach, and vice versa.

**Keywords**  Vector optimization · Set optimization · Set-valued optimization · Pre-order relation

**References**


GENERALIZED INTERVAL-VALUED OPTIMIZATION PROBLEMS

Emrah KARAMAN

Karabük University, Faculty of Science, Department of Mathematics, 78050, Karabük, Turkey
Corresponding Author’s E-mail: e.karaman42@gmail.com

ABSTRACT

We encounter the optimization problems (mathematical programming) in our life. When these problems are expressed mathematically, an objective function emerges. Naturally, solution methods of these problems have been attracted the attention to researchers working in the field such as mathematics, engineering, computer science, economics, business, etc. for years.

Optimization problems are classified according to the objective function. For example, we say that the problem is the interval-valued optimization problem (shortly, interval optimization problem) when the objective function is interval-valued. Interval optimization problems are a generalization of the scalar optimization problems because the interval-valued function is a generalization of the real-valued functions. Also, the interval optimization problems are a special form of the set-valued optimization problems. Order relations defined on intervals are used to solve interval optimization problems. In the literature, these order relations are obtained by using natural order relation on the real numbers [2, 2, 2, 4, 7, 5, 6].

In any vector space, there is no natural order relation such as in real numbers. So, an order relation is defined with the help of a subset of the space in order to compare the vectors. This set is called ordering cone. This order relation is either preorder relation, partial order relation or total order relation according to ordering cone. So, all definitions and results used in vector optimization problems depend on the ordering cone. Also, all order relations, which are defined on vector space, induce an ordering cone. Thus, each order relation in the vector space corresponds to an ordering cone and vice versa. Naturally, minimal and maximal element definitions can also change when ordering cone or order relation change. More information about the vector optimization can be found in [8].

The purpose of this study is to get a general version of interval optimization problems on any real vector space. Thus, intervals are defined by using the ordering cone. So, generalized interval optimization problems will be constructed. Also,
two order relations are introduced to obtain the solution of the interval optimization problems.

**Keywords** Interval-valued optimization · Vector optimization

**References**


ON NEW NARAYANA POLYNOMIALS

Engin ÖZKAN ¹, Bahar KULOĞLU ²

¹Department of Mathematics, Erzincan Binali Yıldırım University, Faculty of Arts and Sciences, Erzincan, Turkey
²Graduate School of Natural and Applied Sciences, Erzincan Binali Yıldırım University, Erzincan, Turkey

Corresponding Author’s E-mail: eozkan@erzincan.edu.tr

ABSTRACT

Some number sequences have a great importance in mathematics and they have many great applications such as calculus, applied mathematics and so on. It is not well known whether Narayana numbers have similar applications. But if we establish a close relationship between the studies and the features of the Fibonacci sequences, we can confidently talk about the application area in many fields, just like Fibonacci sequence.

The Narayana numbers come up as a result of solving the problem of the Narayana’s cow which was given by the Indian mathematician Narayana. This problem is that: A cow produces one calf every year. Beginning in its fourth year, each calf produces one calf at the beginning of each year. How many calves are there altogether after 20 years? [1].

Let n show the year. So, the Narayana problem can be given by the recurrence relation:

\[ N_{n+1} = N_n + N_{n-2}, n \geq 2 \]

where \( N_0 = 0, \ N_1 = 1 \)and \( N_2 = 1 \)[1].

The first few terms of the Narayana numbers are 0,1,1,2,3,4,6,9,13,… This sequence is called Narayana sequence [1]. It is well known that the Narayana numbers are given by Binet’s formula

\[ N_n = \frac{\alpha^{n+1}}{(\alpha - \beta)(\alpha - \gamma)} + \frac{\beta^{n+1}}{(\beta - \alpha)(\beta - \gamma)} + \frac{\gamma^{n+1}}{(\gamma - \alpha)(\gamma - \beta)} \]

where

\[ \alpha = \frac{1}{3} \left( 1 + \sqrt[3]{\frac{2}{29 + 3\sqrt{93}}} + \sqrt[3]{\frac{29 + 3\sqrt{93}}{2}} \right) \]
\[ \beta = \frac{1}{3} \left( 1 - w^3 \sqrt{\frac{2}{29 + 3\sqrt{93}}} + w^2 \sqrt{\frac{29 + 3\sqrt{93}}{2}} \right) \]

and

\[ \gamma = \frac{1}{3} \left( 1 + w^3 \sqrt{\frac{2}{29 + 3\sqrt{93}}} + w^2 \sqrt{\frac{29 + 3\sqrt{93}}{2}} \right) \]

where \( w = \frac{1 + i\sqrt{3}}{2} \) is the primitive cube root of unity [1].

The Fibonacci polynomials were studied in 1883 by E. C. Catalan. The Fibonacci polynomials \( F_n(x) \) are defined by the recurrence relation:

\[ F_n(x) = xF_{n+1}(x) + F_n(x), \quad n \geq 1 \]

with the initial condition \( F_1(x) = 1 \) and \( F_2(x) = x \).

When \( x = 1 \), we get the Fibonacci numbers [7].

We can see some important properties of the Narayana sequence in [2, 10, 1]. The Pell polynomials are defined by

\[ P_n(x) = 2xP_{n-1}(x) + P_{n-2}(x), \quad n \geq 2 \]

where \( P_0(x) = 0 \) and \( P_1(x) = 1 \) [7, 9]. They are related to the Fibonacci polynomials by \( P_n(x) = F_n(2x) \).

In this study we gave a new definition of Narayana. We showed that there is a relationship between the coefficient of the Narayana polynomials and Pascal’s triangle and Pell numbers. Afterwards we also gave some definitions, properties and theorems related to the numbers and the polynomials. We define the Hankel Transform of them.

**Keywords** Narayana sequence · Narayana polynomials · Pascal triangle.

**References**


THE $k$-NARAYANA SEQUENCE

Engin ÖZKAN $^1$, Bahar KULOĞLU $^2$

$^1$Department of Mathematics, Erzincan Binali Yıldırım University, Faculty of Arts and Sciences, Erzincan, Turkey
$^2$Graduate School of Natural and Applied Sciences, Erzincan Binali Yıldırım University, Erzincan, Turkey

Corresponding Author’s E-mail: eozkan@erzincan.edu.tr

ABSTRACT

Narayana was an outstanding Indian mathematician of the XIV century. From him came to us the manuscript "Bidzhahanity" (incomplete), written in the middle of the XIV century. For Narayana was interesting summation of arithmetic series and magic squares. [2]

The Narayana numbers come up as a result of solving the problem of the Narayana’s cow which was given by the Indian mathematician Narayana. This problem is that: A cow produces one calf every year. Beginning in its fourth year, each calf produces one calf at the beginning of each year. How many calves are there altogether after 20 years? [1].

Let $n$ show the year. So, the Narayana problem can be given by the recurrence relation [9]:

$$N_{n+1} = N_n + N_{n-2} , n \geq 2$$

where $N_0 = 0$, $N_1 = 1$ and $N_2 = 1$

The first few terms of the Narayana numbers are 0,1,1,2,3,4,6,9,13,\ldots This sequence is called Narayana sequence [8].

The generalized Narayana sequence especially, for any nonzero integer $k$, the $k$-Narayana sequence is defined in [8] by the recurrence relation:

$$N_{k,n} = kN_{k,n-1} + N_{k,n-3}$$

with the initial values $N_{k,0} = 0$, $N_{k,1} = 1$ and $N_{k,2} = k$.

The first few terms are:

$$0, 1, k, k^2, k^3 + 1, k^4 + 2k, k^5 + 3k^2, \ldots$$

It is well known that the $k$—Narayana numbers are given by Binet’s formula

$$N_{k,n} = \frac{\alpha_n^k}{(\alpha_k - \beta_k)(\alpha_k - \gamma_k)} + \frac{\beta_n^k}{(\beta_k - \alpha_k)(\beta_k - \gamma_k)} + \frac{\gamma_n^k}{(\gamma_k - \alpha_k)(\gamma_k - \beta_k)}$$
where

\[
\alpha_k = \frac{1}{3} \left( k + k^2 \sqrt[3]{\frac{2}{27 + 2k^3 + 3\sqrt{81 + 12k^3}}} + \sqrt[3]{27 + 2k^3 + 3\sqrt{81 + 12k^3}} \right)
\]

\[
\beta_k = \frac{1}{3} \left( k - wk^2 \sqrt[3]{\frac{2}{27 + 2k^3 + 3\sqrt{81 + 12k^3}}} + w^2 \sqrt[3]{27 + 2k^3 + 3\sqrt{81 + 12k^3}} \right)
\]

and

\[
\gamma_k = \frac{1}{3} \left( k + w^2k^2 \sqrt[3]{\frac{2}{27 + 2k^3 + 3\sqrt{81 + 12k^3}}} + w^3 \sqrt[3]{27 + 2k^3 + 3\sqrt{81 + 12k^3}} \right)
\]

and

\[
w = \frac{1 + \sqrt{3}i}{2}\]

is the primitive cube root of unity.

Characteristic equation of the \(k\)-Narayana numbers is that

\[x^3 - kx^2 - 1 = 0\]

For any integer number \(k\), \((k \neq 0)\), we have the following matrixes

\[Q_k = \begin{pmatrix} k & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\]

and for \(n \geq 3\),

\[(Q_k)^n = \begin{pmatrix} N_{k,n+1} & N_{k,n-1} & N_{k,n} \\ N_{k,n} & N_{k,n-2} & N_{k,n-1} \\ N_{k,n-1} & N_{k,n-3} & N_{k,n-2} \end{pmatrix}\]

Then \(Q_k\) is a generating matrix of the \(k\)-Narayana sequence.

The general \(k\)-Fibonacci sequence generalizes, between others, both the classical Fibonacci sequence and the Pell sequence.

We give some properties and theorems related to the \(k\)-Narayana numbers. We prove the Cassini identity and a limit of the \(k\)-Narayana sequence. We calculate the determinant of the \(k\)-Narayana numbers and show that their determinant are fixed terms. Further, we give sum of the first terms of the \(k\)-Narayana sequence and we deduce many properties of the \(k\)-Narayana sequence.

**Keywords** k-Narayana sequence · Catalan Transform · Hankel Transform

**References**


In this paper, it is studied at the group of $\hat{\Gamma}_{0,n}(N)$. It is generated by the group $\Gamma_{0,n}(N)$ with respect to reflection transformation $R(z) = -\overline{z}$. Firstly, it is obtained that the group does not contain second and third degree elliptic element under some conditions. Secondly, it is defined the set of $\hat{Q}(N)$. Thirdly, an equivalence relation is established for the group of $\hat{\Gamma}_{0,n}(N)$ in $\hat{Q}(N)$. Then, it is researched the edge conditions in the graph $G^*_{u,N}$, which is obtained by the equivalence relation. Fourth, it is proved that the graph $F^*_{u,N}$ does not contain a triangle. $F^*_{u,N}$ is a subgraph which is edges of graph $G^*_{u,N}$ in the block $[\infty] = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$. Finally, it is expressed and proved that the graph $F^*_{u,N}$ is a forest under some conditions.

**Keywords** Suborbital graph · Elliptic element · Edges of graph · Forest

**References**


ABSTRACT

Transportation of a hazardous material such as explosives and ammunition can be a target for an enemy attack. While a competent authority for hazardous material transportation (HMT) aims to minimize the total risk to public or environment, an attacker might attempt to maximize it by destroying some specific segments (arcs) on the transportation network. The problem can be designed as a bi-level network interdiction model [1,2,3]. Generally speaking, there are two players in a network interdiction problem. Network owner, who naturally tries to run the network optimally, is the defending side (defender). The opponent, trying to impede the network process on the other side, is the attacking one (interdictor/attacker). With limited resources at hand, the attacker inflicts some negative effects on the network arcs so that the objective of the network owner is maximally deteriorated. In this study, we first develop a bi-level max-min HMT attack model (Bi-HMTA) considering that the network is liable to one or more enemy attacks depending on the attack resources available. Then to solve the problem, we convert the model Bi-HMTA into an ordinary mixed integer programming (MIP) model (HMTA) using duality theorem, integer and combinatorial optimization techniques [4,5,6]. Finally, we test the resulting model HMTA using different size networks with various attack resources. The results reveal out the arcs that are most liable to an enemy attack as well as the ensuing increase in the total risk of transportation. In our specific application, we observe that roads between train unload stations and demand points are most likely targets for an attack. The total risk substantially increases depending on the available attack resources. The decision makers can get use of such results in analyzing the risk of transportation through a specific path and develop their own strategies accordingly to decrease the risk they encounter in transporting the hazardous materials.
Keywords Bi-Level Modeling · Integer Programming · Network Modeling · Network Interdiction · Hazardous Materials Transportation · Risk Analysis

References

THE INTRINSIC METRIC ON THE SCALE IRREGULAR SIERPINSKI GASKET $SG(2, 3)$

Fatma Diğdem KOPARAL$^1$, Yunus ÖZDEMİR$^1$

$^1$Eskişehir Technical University, Department of Mathematics, 26470, Eskişehir, Turkey
Corresponding Author’s E-mail: fdyildirim@eskisehir.edu.tr
This work is supported by the Eskişehir Technical University Research Fund Under Contract 19ADP113.

ABSTRACT

The Sierpinski Gasket is the well-known self-similar set in fractal geometry. There exist several generalizations of this set in the literature such as regular Sierpinski Gaskets, mod-$p$ Sierpinski Gaskets (or Pascal-Sierpinski Gaskets), the Sierpinski pedal triangles and etc. A scale irregular Sierpinski Gasket formed by regular Sierpinski Gasket also can be thought as an another generalization (see [1] for more details).

On the other hand, expressing the intrinsic metric and determining the geodesics of a self-similar set such as the (classical) Sierpinski Gasket ($SG(2)$), the Vicsek fractal, the Sierpinski carpet etc. has been studied in the last decades by using different tools and techniques (see [2] for example). In [3], the intrinsic metric of the (classical) Sierpinski Gasket ($SG(2)$) is expressed via the code representations of the points, and the authors show that there exist at most five geodesics between two different points. In [4], for the regular Sierpinski Gasket $SG(3)$, the author gives an expression of the intrinsic metric via code representation of the points and show that there can be infinitely many geodesics between two different points.

In this work, we express the intrinsic metric formula of the scale irregular Sierpinski Gasket $SG(2, 3)$ formed by $SG(2)$ and $SG(3)$. To express the intrinsic metric of $SG(2, 3)$, we use the code representations of the points (as done in the cases $SG(2)$ and $SG(3)$). Using the previous results, we also investigate the geodesics in this irregular case.

Keywords  Sierpinski Gasket · Scale irregular Sierpinski Gasket · Intrinsic metric · Geodesic

References


NON-ROTATING FRAMES AND ITS APPLICATIONS

Fatma KARAKUŞ

Sinop University, Faculty of Sciences and Arts, Department of Mathematics, Sinop, Turkey
Corresponding Author’s E-mail: fkarakus@sinop.edu.tr

ABSTRACT

In this paper, I will explain Fermi-Walker derivative and non-rotating frame concepts for some frames in Euclidean space, Minkowski space, Dual Space, Dual Lorentzian space. All non-rotating frame conditions will be analyzed for Frenet frame, Bishop frame and Darboux frame in Euclidean space, Minkowski space, Dual space, Dual Lorentzian space. Then, I will explain non-rotating frame notions for three-dimensional Lie group with left-invariant metric. I will generalize the results known for the case of three-dimensional Lie group with the bi-invariant metric. I will define Bishop frame (relatively parallel adapted frame accordance with dot-Frenet frame). After that, I will explain that the Bishop frame and Frenet frame whether they are a non-rotating frame or not for three-dimensional Lie group with left-invariant metric. I will give some examples for the non-rotating frames applications.

Keywords Fermi-Walker Derivative, Non-Rotating Frame, Euclidean space, Dual Space, Minkowski Space, 3-Dimensional Lie Group

Acknowledgements. This research has been supported by the Scientific and Technological Research Council of Turkey (TUBITAK).

References


FERMI-WALKER PARALLELISM AND ITS APPLICATIONS

Fatma KARAKUŞ

Sinop University, Faculty of Sciences and Arts, Department of Mathematics, Sinop, Turkey
Corresponding Author’s E-mail: fkarakus@sinop.edu.tr

ABSTRACT

In this paper, I will explain Fermi-Walker derivative and Fermi-Walker parallelism concepts in Euclidean space, Minkowski space, Dual Space, Dual Lorentzian space. I will explain the necessary conditions to be Fermi-Walker parallelism along any curve for any vector field in Euclidean space, Minkowski space, Dual space, Dual Lorentzian space. Also, I will explain Fermi-Walker parallelism concept for three-dimensional Lie group with left-invariant metric and I will generalize the results known for the case of three-dimensional Lie group with the bi-invariant metric. I will give some examples for the Fermi-Walker parallel vector fields applications.

Keywords Fermi-Walker Derivative, Fermi-Walker Parallelism, Space Curves, Plane Curves, Line of Curvature, Geodesic Curves

Acknowledgements. This research has been supported by the Scientific and Technological Research Council of Turkey (TUBITAK).

References

DYNAMICS OF A MODIFIED A DISCRETE-T İME PREDATOR- PREY MODEL WITH ALLEE EFFECT

Figen KANGALGİL¹, Seval IŞIK²

¹Dokuz Eylül University, Bergama Vocational High School, 35700, Izmir, Turkey
²Sivas Cumhuriyet University, Faculty of Education, Department of Mathematics and Science Education, 58140, Sivas, Turkey

Corresponding Author’s E-mail: skaracan@cumhuriyet.edu.tr

ABSTRACT

The dynamics of interaction between predators and preys have great interested in ecology and mathematical biology. Differential and difference equations have been used to investigate a wide range of population models. The populations models governed by differential equations studied extensively by many researchers [1,2] and the reference therein. However, in recent years considerable number articles in literature discussed the dynamic of the discretization of predator prey models governed by differential equations [3,4]. Because the discrete-time models present dynamics consistency according to continuous-time. In addition, the discrete-time population models are suitable for non-overlapping generations and are appropriate to describe the nonlinear dynamics and possibly their chaotic behaviour.

Bifurcation theory is research field that analyzes the change of dynamical models with respect to a control parameter. So, the behaviour the model according to a control parameter is observed. There are many studies on bifurcation analysis [5].

The Allee effect, a reduction of the per capita growth rate of a population of biological species at densities smaller than a critical value, was first introduced by Allee in 1931 [7]. Although there are many extensive researches on Allee effect, many articles have not addressed the Allee effect with focus on bifurcation analysis of discrete-time predator prey model [8]. Thus, in this work it is investigated that bifurcation and stability analysis of a modified a discrete-time predator prey model which developed from the inclusion of Allee effect in predator population.

The objective of this work is to investigate a discrete-time predator- prey model with Allee effect in predator species. The Euler method is constructed to transform the continuous time predator prey model with Allee effect into the discrete-time model. The existence of coexistence fixed point of the considered model is studied. The parametric conditions for local asymptotic stability of fixed point of the discrete-time model is presented. Moreover, it is proved that model undergoes
Neimark-Sacker bifurcation for small range of parameters by using bifurcation theory. Finally, some numerical simulations have been carried out in order to support of our analytical findings.

**Keywords** Stability · Bifurcation · Prey-Predator Model

**References**


NUMERICAL METHODS FOR A NONLOCAL PERIDYNAMIC MODEL

Giuseppe Maria COCLITE\textsuperscript{1}, Alessandro FANIZZI\textsuperscript{2}, Luciano LOPEZ\textsuperscript{2}, Francesco MADDALENA\textsuperscript{1}, Sabrina Francesca PELLEGRINO\textsuperscript{2}

\textsuperscript{1}Department of Mechanics, Mathematics and Management, Polytechnic of Bari, Via E. Orabona 4, 70125 Bari, Italy
\textsuperscript{2}Department of Mathematics, University of Bari, Via E. Orabona 4, 70125 Bari, Italy

Corresponding Author’s E-mail: sabrina.pellegrino@uniba.it

ABSTRACT

We consider the peridynamic equation of motion, which is described by a second order in time partial integro-differential equation. This equation has recently received great attention in several fields of Engineering because seems to provide an effective approach to modeling mechanical systems avoiding spatial discontinuous derivatives and body singularities. In particular, we consider the linear model of peridynamics in a one-dimensional spatial domain. We review some numerical techniques to solve this equation and propose some new computational methods of higher order in space. We discuss the implementation of a spectral method for the spatial discretization of the linear problem. Several numerical tests are given in order to validate our results.

Keywords Peridynamics · Quadrature formula · Spectral methods · Trigonometric time discretization

References

DOUBLE-DIFFUSION INSTABILITY: A FINGERING CONVECTION MODEL FOR OCEANIC STAIRCASE FORMATION

Giuseppe Maria COCLITE¹, Francesco PAPARELLA², Sabrina Francesca PELLEGRINO³

¹Department of Mechanics, Mathematics and Management, Polytechnic of Bari, Via E. Orabona 4, 70125 Bari, Italy
²Department of Mathematics, New York University in Abu Dhabi, Saadiyat Island, P.O. Box 129188, Abu Dhabi, United Arab Emirates
³Department of Mathematics, University of Bari, Via E. Orabona 4, 70125 Bari, Italy

Corresponding Author’s E-mail: sabrina.pellegrino@uniba.it

ABSTRACT

Fingering convection is a mechanism of double-diffusion which occurs in fluids where density is controlled by two buoyancy-changing scalars, temperature and salinity, that diffuse at different rates. This set-up allows the convective cells to assume the shape of tall finger-like structures called “salt fingers”.

We consider a system of nonlinear degenerate parabolic equations describing the phenomenon in terms of horizontally averaged kinetic energy field and buoyancy. We prove the existence of weak solutions for such system and discuss their stability and asymptotic properties.

Keywords Salt fingers · Water waves · Degenerate parabolic equations · Weak solutions

References

EIGENVALUES OF THE SELFADJOINT SCHRÖDINGER OPERATOR ON NON-COMPACT STAR GRAPH

Gökhan MUTLU

Gazi University, Faculty of Science, Department of Mathematics
Corresponding Author’s E-mail: gmutlu@gazi.edu.tr

ABSTRACT

The point spectrum of the non-compact quantum star graph $\Gamma$ with $n$ edges of infinite length has been investigated. We assume the Hamiltonian on this non-compact graph acts as Schrödinger operator $-f'' + Q(x)f$ on each edge where the potential $Q(x)$ is a real-valued function which is Lebesgue integrable on $(0, \infty)$ and

$$\int_0^\infty (1 + x) |Q(x)| \, dx < \infty,$$

holds. We assume that the central vertex $v$ possesses the most general selfadjoint vertex conditions

$$A_v F(v) + B_v F'(v) = 0,$$

$$\text{Rank} (A_v \mid B_v) = n,$$

$$A_v B_v^* - B_v A_v^* = 0,$$

where $A_v$ and $B_v$ are $n \times n$ matrices and $"*"$ denotes the matrix adjoint.

Quantum graphs have many applications in physics and engineering. Quantum graphs enable us to model systems in mathematics, physics, chemistry, and engineering to analyze phenomena such as the free-electron theory of conjugated molecules, quantum wires, dynamical systems, photonic crystals, thin waveguides, and many other. As a result, there is a remarkable interest in this field nowadays. Most of the studies concern the negative second derivative operator $-f''$ i.e. $Q(x) \equiv 0$ acting on the edges of graphs [1, 2, 3]. The Schrödinger operator on compact quantum graphs have been also considered [4, 5].

Obviously, the Hamiltonian on $\Gamma$ is selfadjoint. Therefore, all eigenvalues are real. We relate this scattering model on non-compact star graph with the matrix Schrödinger operator on the half-line

$$H_{A,B} := -y'' + V(x)y, \quad x \in (0, \infty),$$
where the potential $V(x)$ is an $n \times n$ matrix-valued function such that $V(x)^* = V(x)$ for each $x \in (0, \infty)$ and
\[
\int_0^\infty (1 + x) \|V(x)\| dx < \infty,
\]
holds where $\| \cdot \|$ denotes any of the equivalent matrix norms. The boundary condition at the origin is assumed to be the most general selfadjoint boundary conditions
\[
Ay(0) + By'(0) = 0, \\
\text{Rank}(A \mid B) = n, \\
AB^* - BA^* = 0,
\]
where $A$ and $B$ are $n \times n$ matrices.

We observe that the Hamiltonian on non-compact star graph and the matrix Schrödinger operator $H_{A,B}$ on the half-line are isospectral. We find the resolvent of $H_{A,B}$ and show that the eigenvalues of the non-compact star graph coincides with the zeros of the Jost function in the upper half-plane. We show that the multiplicity of each eigenvalue is finite and this multiplicity is equal to the multiplicity of the corresponding zero of the Jost function. Finally, we prove that the non-compact star graph has a finite number of eigenvalues under the condition (1).

**Keywords** Non-compact star graph · Eigenvalues · Matrix Schrödinger operator

**References**


EVOLUTE OFFSET OF NON-CYLINDIRICAL RULED SURFACES WITH B-DARBOUX FRAME

Gül Uğur KAYMANLI

Department of Mathematics, Cankiri Karatekin University, Cankiri, 18100, Turkey

Corresponding Author’s E-mail: gulugurk@karatekin.edu.tr

ABSTRACT

A ruled surface in three dimensional space is a surface which can be described as the set of points swept by a moving straight line. By using the directions, many offset of this surface (or curve) such as Bertrand, involute-evolute, Mannheim and Smarandache have been defined. Both offsets surfaces and ruled surfaces are widely used in kinematics, mechanism, Computer-Aided Geometric Design (CAGD) and geometric modelling. Offsets of ruled surfaces are generally more complex than their progenitor ruled surface. Because of this offsets surfaces (curves) are examined in terms of the properties of the base surface (curve) is important. In this work, we introduce the evolute offset of non-cylindrical ruled surfaces with B-Darboux frame. Some geometric properties of this evolute offset of non-cylindrical ruled surface are studied. That is, we examine the striction curve, distribution parameter and the developability of evolute offset of given ruled surfaces in terms of B-Darboux frame in three dimensional Euclidean space.

Keywords B-Darboux Frame · Curvatures · Evolute Offset · Ruled Surface

References


THE RESTRICTIVE SETS AND THEIR APPLICATIONS TO SOFT GROUP THEORY

Hakan AYKUT$^1$, Akın Osman ATAGÜN$^2$

$^{1,2}$Kırşehir Ahi Evran University, Turkey

Corresponding Author's E-mail: aosman.atagun@ahievran.edu.tr

ABSTRACT

Molodtsov [12] introduced the soft set theory to give a new approach to solve problems including uncertainties. He gave many applications of this theory in many areas such as information systems, probability and measurement theory. Applications of soft set theory are progressing rapidly also in the areas decision making, optimization theory, algebraic structures. This study includes a novel algebraic application of soft set theory using a nonempty subset of initial universe set. We call this set "generator set". In this paper, firstly four different restrictions, $\alpha$ – including, $\alpha$ – covered, $\alpha$ – intersection and $\alpha$ – union sets of a soft set over an initial universe $U$ obtained by using a non-empty subset $\alpha$ of $U$ are examined, in detail. We investigate some useful novel properties of these four restrictive sets according to soft set operations throughout. Firstly, we give some properties of the restrictive sets according to soft subset $\subseteq$. Then, some properties of the restrictive sets according to the operation soft union "$\bar{U}$", soft restricted union "$\bar{\cup}_R$", soft extended intersection "$\bar{\cap}_e$", AND operation "$\bar{\land}$", OR operation "$\bar{\lor}$" and restricted difference "$\sim_R$" are given. Finally, some properties of soft sets that are restricted by $\alpha$ – intersection and $\alpha$ – union are investigated. We also give examples of the properties of restrictive sets. Then an application of $\alpha$ – intersection sets over a soft intersection groups is given. Here, the novel concept of the soft int-subgroup generated by $\alpha \subseteq U$ of a group is introduced and some properties are given. Then we obtained a relationship between a subgroup of a group and a soft subgroup of the same group. Also we examined count of generators of a soft intersection groups. Furthermore we investigated connections of generated sets of same group. The generators of soft intersections and products of soft int-subgroups are given. Some related properties about generators of soft int-subgroups are investigated and illustrated by several examples.

Keywords Soft set · Soft int-group · Restrictive set · Generator of a group
References


Mathematical Model of Extinction of
*Rastrelliger brachysoma*, harvesting and control

Inthira CHAIYA¹, Kamonchat TRACHOO¹, Din PRATHUMWAN²

¹Department of Mathematics, Faculty of Science, Mahasarakham University, Mahasarakham 44150, Thailand
²Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand
Corresponding Author’s E-mail: dinpr@kku.ac.th

Abstract

In this work, we proposed a mathematical model of the population density of Indo-Pacific mackerel and the population density of small fishes based on impulsive fishery. The developed impulsive mathematical model was analyzed theoretically in order to obtain the conditions on the parameters of the model for which Indo-Pacific mackerel (*Rastrelliger brachysoma*) conservation might be expected. Numerical results were also carried out to confirm our theoretical predictions. The results showed that the duration and quantity of fisheries were the keys to prevent the extinction of Indo-Pacific mackerel.

Keywords  Impulsive model · Indo-Pacific mackerel · Stability · Harvesting control

References

SOME APPLICATIONS OF EIGENVALUE AND EIGENVECTORS IN ENGINEERING

Ion MIERLUS MAZILU 1, Fatih YILMAZ 2

1 Technical University of Civil Engineering Bucharest, Bucharest, Romania
2 Ankara Haci Bayram Veli University, Ankara, Turkey

Corresponding Author’s E-mail: ion.mierlusmazilu@utcb.ro

ABSTRACT

In engineering and science, many matrix applications benefit from eigenvalue and eigenvector. For any \( n \)-square matrix \( A \), the special numbers which are called as eigenvalues and some special vectors which are called as eigenvectors have great importance. Here we exemplified their applications in stretching of an elastic membrane, population growth, using eigenvalues and eigenvectors to study vibrations, Google’s page rank.

Keywords: Eigenvalue · Eigenvector · Vibration

References

GENERALIZED QUASI-EINSTEIN NORMAL METRIC CONTACT PAIR MANIFOLDS

İnan ÜNAL

Department of Computer Engineering, Munzur University, Tunceli, Turkey
Corresponding Author’s E-mail: inanunal@munzur.edu.tr

ABSTRACT

In this study, we investigate generalized quasi-Einstein structure for normal metric contact pair manifolds. We obtain some elementary properties and we give the characterizations of generalized quasi-Einstein normal metric contact pair manifolds. We consider this structure under some curvature conditions and we give some related results.

Keywords generalized quasi-Einstein · metric contact pairs · Generalized quasi-constant curvature · $\mathcal{M}$—projective curvature tensor

References


A NEW APPROACH TO GENERALIZED CANTOR SET IN FRACTAL GEOMETRY

İpek Ebru KARAÇAY¹, Salim YÜCE¹

¹Yıldız Technical University, Faculty of Art and Sciences, Department of Mathematics, 34220, İstanbul, Turkey
Corresponding Author’s E-mail: ipekebrukaracay@gmail.com (https://orcid.org/0000-0002-5289-6457), sayuce@yildiz.edu.tr (https://orcid.org/0000-0002-8296-6495)

ABSTRACT

Many studies have been done about the fractal geometry from past to present. However, the meaningful development of fractal geometry was taken place with increasing of the computer science. Fractal geometry emerged when French mathematician Mandelbrot discovered the Mandelbrot set by giving complex number values to quadratic polynomials, which are described by his name and applied this method at each step continuously infinitely many times, [1]. Before the development of computer technologies, fractal structures such as Cantor Set and Sierpinski Gasket had been identified without being called fractals. Cantor set was first introduced by G.Cantor in 1883, [2]. It is defined by [0, 1] closed interval, which is a subset of the set of real numbers, being reduced by 1/3 reduction rate infinitely, [2, 3]. When the length calculation of this structure is made, the result is zero and it is concluded that the concept of length does not indicate a characteristic feature for this fractal structure, [3]. Then, the concepts of self-similarity and fractal dimension, which are two features specific to fractal structures, were defined. Fractals are created with iterated function systems by making use of self-similarity feature, [3, 4]. Generalized Cantor set is defined for [0, 1] closed interval, which is a subset of the set of real numbers, [5]. For this structure, length and size calculations were made and also fractal structure was created with iterated function systems, [5, 6, 7, 8]. In daily life, fractal geometry has been used in many fields such as architecture, art and medicine.

In this study, Cantor set is defined for the [a, b] closed interval, which is a subset of the set of real numbers. The interval of [a, b] is divided into (2n + 1) parts, creating a series that decreases by 1/(2n+1) and this series goes to infinitely, moreover the result is zero when the length is calculated for the interval [a, b]. Dimension calculation was made for this interval and fractal structure that was obtained with iterated function systems. In addition, [a, b] interval was examined on the curve. The length of the fractal curve obtained in this range was found to be zero and the study was completed by giving the circle example.
Keywords Cantor set · Fractal · Iterated Function System

References

NONLINEAR APPROXIMATION IN DISCRETE OPERATORS OF SAMPLING-TYPE

İsmail ASLAN

Hacettepe University, Department of Mathematics, Ankara, Turkey
Corresponding Author’s E-mail: ismail-aslan@hacettepe.edu.tr

ABSTRACT

In this presentation, our aim is to construct a nonlinear form of sampling type discrete operators, given in [1]. These types of operators have close relationships with generalized sampling series. Taking some specific kernels, it is possible to obtain classical generalized sampling series and nonlinear form of these ones from these operators. On the other hand, summability methods are a common method to get over the lack of convergence. Especially A-summability (see [4, 5]) is fairly general. Beside the classical convergence, it includes many well-known methods, such as Cesaro method, almost convergence and order summability. It is also possible to accelerate the rate of convergences. In the literature, although there exist many applications of this method onto the linear operators [7], there are very few works on nonlinear ones ([2, 3]).

In this study, we also improve discrete operators by using summability method under some generalizations of approximate identities. Considering both the usual supremum norm and convergence in \( \varphi \)-variation, introduced by Musielak and Orlicz in [6], approximation properties of these operators are investigated. While doing this, uniformly and absolutely continuous functions are taken into account respectively. In addition, rates of approximation are evaluated by means of some suitable Lipschitz classes of continuous functions. Approximation in N-dimensional case under supremum norm is also obtained. Thanks to summability method, we show that these approximations are still valid under different convergence methods such as Cesaro’s method, almost convergence method and etc. Finally, we illustrate our theorems and estimations with nontrivial applications. Hence, we clearly show why we need this study.

Acknowledgements. This study is supported by TÜBİTAK 3501 Career Development Program, Project ID: 119F262.

Keywords Sampling Operators · Nonlinear Operators · Summability Method
References


THE WAY OF ASSESSMENT OF MATHEMATICAL COMPETENCIES IN RULES_MATH PROJECT

Jana GABKOVÁ¹, Peter LETAVAJ²

¹,²Slovak University of Technology, Faculty of Mechanical Engineering

Corresponding Author’s E-mail: peter.letavaj@stuba.sk

ABSTRACT

The main goal of the project RULES_MATH was to design the standards for assessment of mathematical competence at technical bachelor studies via contents models. FME STU as one of the partners of the project prepared testing models in the topics Analysis and Calculus and Probability and Statistics. The models were designed with respect to main pedagogical aim, to develop and to assess the mathematical competency through mutually overlapping eight competencies elaborated by SEFI MWG in the Framework [1] and previously introduced by Danish KOM project [2]. We will deal with two guidelines covering the learning outcomes in each model. The mathematical competencies were determined for each learning outcome in tables: Learning outcomes with degree of coverage of competencies involved in this assessment activity. With respect to scope and curricula, we prepared tests, by which the problems in guidelines were verified at FME STU: in the topic Analysis and Calculus at bachelor study programmes in subject Mathematics I for the first year of the study and in the topic Probability and Statistics in compulsorily elective subject Basics of Statistical Analysis for the second year of the study. Levels of acquisition of mathematical competencies were determined by means of given assessment tests. Since not every competence was present in the test with the same importance, it was necessary to determine all competencies covered by the test. In this article, we introduce the process of testing the acquirement of mathematical competencies covered by the test and its results. The main idea for assessment of acquirement of the mathematical competencies is based on division of the points amount achieved by a student in the test into problems/subproblems/subtopics covering the corresponding competencies. After that, these points are distributed on particular competencies in given learning outcome following the “table of competencies” built by teacher for each test. It was needed to norm the resulting values of acquirement of competence in order to analyse it correctly. It means we were supposed to compare these values with maximal possible achievable value for each
competence. Data we obtained were quantitatively processed by program developed in Wolfram Mathematica software. The two input tables and one vector were transformed into one resulting table. The first input table contained the points acquired by each student in each test problem. The second input table contained the importance of each competence in each test problem. Entries of the input vectors were the maximal point amount, that one could earn in each test problem. Entries in the output table give the acquirement of each competence for each student. Consequently, these results (this output table) were statistically processed by software Statgraphics, in order to obtain quantitative analysis of acquired mathematical competencies.

**Keywords** Engineering · Mathematics · Mathematical competence · Assessment · A competencies-based methodology · Rules_Math project

**References**


NONLINEAR ANALYSIS OF HEART RATE DYNAMICS TO ESTIMATE THE RISK OF CARDIOVASCULAR EVENTS IN HYPERTENSIVE PATIENTS

José María LOPEZ-BELINCHON\textsuperscript{1}, Miguel Ángel LOPEZ GUERRERO\textsuperscript{2}, Raúl Alcaraz MARTINEZ\textsuperscript{3}

\textsuperscript{1}Research Group in Dynamical Systems, University of Castilla-La Mancha (UCLM), 16071 Cuenca, Spain
\textsuperscript{2}Polytechnic School of Cuenca, Department of Mathematics and Institute of Applied Mathematics in Science and Engineering (IMACI), University of Castilla-La Mancha (UCLM), 16071 Cuenca, Spain
\textsuperscript{3}Research Group in Electronic, Biomedical and Telecommunication Engineering, University of Castilla-La Mancha (UCLM), 16071 Cuenca, Spain

Corresponding Author’s E-mail: joselopezaguadulce@gmail.com

ABSTRACT

In the last years many theoretical and experimental studies have suggested that most physiological systems are complex and can be better characterized as complex dynamical processes [1]. In fact, it is nowadays well-established the hypothesis that when a physiological system became less complex, it is less adaptable to external changes, and hence is prone to suffering from a disease [1, 2]. Accordingly, a broad variety of physiological time series have been recently analyzed from a nonlinear point of view to improve current understanding of the mechanisms triggering different pathologies [3].

Although nonlinear analysis has been widely applied to characterize cardiac dynamics in a variety of scenarios [3], this kind of study has not still been considered to estimate the risk of cardiovascular event in hypertensive patients [4]. Hence, the main goal of the present study is to analyze the ability of different complexity-based metrics to anticipate patients at high risk of suffering a cardiovascular event (i.e., myocardial infarction, stroke, syncopal event, etc.) during a follow-up of one year. Indices such as sample entropy, fuzzy entropy, spectral entropy, and C0 complexity have been computed from the heart rate time series derived from 24-hour electrocardiogram (ECG) recordings acquired for 139 hypertensive patients.

Some of the analyzed nonlinear indices have been able to successfully identify about 70\% of those patients who suffered one or more events within the follow-up. Moreover, this diagnostic accuracy has increased to about 75\% by combining several metrics with different matching learning classifiers. These results overcome
those reported by common clinical indicators, such as echocargraphic parameters, thus opening a new door to improve the early diagnosis of hypertensive patients at high-risk to develop future vascular and cardiovascular events.

**Keywords** Nonlinear analysis · Heart rate variability · Classification

**References**


This paper represents the new novel technique of Natural Transform Adomian Decomposition Method (NTADTM) to get real and imaginary option prices of two stocks in form of analytic infinite series by solving Schrödinger Black Scholes time fractional ordered PDE consisting two different stocks. For that reason, the approach is discovered to be appropriate for the models that can be expressed as partial differential equations of integer and fractional orders, subjected to preliminary or boundary situations. This technique is the combination of Natural Transform and Adomian Decomposition Method.

**Keywords** Natural Integrals Transform 1 · Adomian Decomposition Method · Time fractional order Schrödinger black Scholes PDE

**References**


THE STUDY OF AXIAL LOAD IN THE LUMBAR SPINE OF PATIENTS WITH LUMBAR SPINAL STENOSIS BY USING FINITE ELEMENT METHOD

Kamonchat TRACHOO¹, Inthira CHAIYA¹, Din PRATHUMWAN²

¹Department of Mathematics, Faculty of Science, Mahasarakham University, Mahasarakham 44150, Thailand
²Department of Mathematics, Faculty of Science, Khon Kaen University, Khon Kaen 40002, Thailand

Corresponding Author's E-mail: dinpr@kku.ac.th

ABSTRACT

Lumbar spinal stenosis is caused by stenosis of the vertebral cavity which may be caused by thicker bone, tendon thicker, etc. In this paper, a mathematical model is developed to simulate a two-dimensional Lumbar spine (L3), taking into account the stress distribution and total displacement during the axial load of the lumbar spine in lumbar spinal stenosis patients and people without the disease. A realistic domain is created based on CT scan data. Using the finite element method, the total displacement and the Von Misses stress is carried out. The influences of lumbar spinal stenosis patients and people without disease during three different cases of bending down are investigated. The advantage of this research has utilized surgery involved with lumbar spinal stenosis.

Keywords Lumbar spinal stenosis · Finite element method · Numerical simulation

References

IMPROVING ENGINEERING THERMODYNAMICS LEARNING WITH MATHEMATICA

María Jesús SANTOS¹, Alejandro MEDINA¹, José Miguel Mateos ROCO¹, Araceli QUEIRUGA-DIOS²

¹Dpt. of Applied Physics and Research Institute for Fundamental Physics and Mathematics, University of Salamanca, Spain
²Dpt. of Applied Mathematics, University of Salamanca, Spain

Corresponding Author’s E-mail: smjesus@usal.es

ABSTRACT

Sophomore students from the Chemical Engineering Degree are involved in a Mathematics course during the first semester and an Engineering Thermodynamics course during the second one. When they participate in the latter they are already familiar with mathematical software to solve numerical methods problems, including non-linear equations, interpolation, or differential equations. We present in this paper, on the one hand, some of the materials elaborated in both courses with the Mathematica® tool, and on the other, the didactic organization of the Engineering Thermodynamics course. The objective of the experience is to increase the inter-relationship between different subjects, to promote transversal skills, and to make the subject closer to the real working procedure the students will find in their future working life [1]. The satisfactory results of the experience are exposed in this work.

Keywords Engineering Education · Engineering Thermodynamics · Numerical and computational Methods

References

AN APPLICATION OF DOUBLE SUMUĐU TRANSFORM FOR SOLVING TELEGRAPH EQUATION

Maria AYDIN, Haldun Alpaslan PEKER

1 Graduate School of Natural and Applied Sciences, Selcuk University, Konya, Turkey
2 Department of Mathematics, Faculty of Science, Selcuk University, Konya, Turkey

Corresponding Author’s E-mail: toermaria@gmail.com, ORCID: 0000-0002-1862-2911

ABSTRACT

The modelling of the most of the physical and biological phenomena, engineering models, real life problems are expressed mathematically in terms of differential equations. There are so many different techniques to solve differential equations. Integral transform techniques such as Laplace, Fourier, etc. are extensively applied in theory and application [1]. There are a lot of work on the theory and applications of the integral transformations. Sumudu transform, which is one of the integral transforms, is not only having very special and useful properties but also be able to help with intricate applications in science and engineering [2]. Some of these applications can be found in [3]. Although the Sumudu transform is the theoretical dual to the Laplace transform, the Sumudu transform rivals it in problem solving. Besides this, the Sumudu transform is also widely and extensively used for solving differential equations. This transform is one of the very convenient mathematical tools for solving differential equations. The definition, properties and applications of the Sumudu transform to differential equations were described in [4]. The main advantage of the Sumudu transform is the fact that it may be used to solve the problems without resorting to a new frequency domain [5].

In recent years, many various methods such as double Laplace transform, double Sumudu transform introduced by many researchers to find the solution of partial differential equations. The definition, properties and applications of the double Sumudu transform were described in [5].

In this study, we believe for the first time, the exact solution of general linear telegraph equation which appear in the propagation of electrical signals along a telegraph line, digital image processing, telecommunication, signals and systems is obtained by double Sumudu transform. On the other hand, in addition to these, this study is a part of the M.Sc. Thesis of the first author [6].
Keywords  Sumudu Transform · Double Sumudu Transform · Telegraph Equation

References

TRIPLE SUMUDU TRANSFORM AND ITS APPLICATION FOR SOLVING VOLterra INTEGRO-PARTIAL DIFFERENTIAL EQUATION

Maria AYDIN 1, Haldun Alpaslan PEKER 2

1 Graduate School of Natural and Applied Sciences, Selcuk University, Konya, Turkey
2 Department of Mathematics, Faculty of Science, Selcuk University, Konya, Turkey

Corresponding Author’s E-mail: tomermaria@gmail.com, ORCID: 0000-0002-1862-2911

ABSTRACT

The theory and application of ordinary and partial differential equations play an important role in the modelling of most of the physical phenomena, biological models, engineering sciences, real life problems. Thus, ordinary and partial differential equations are significant both in mathematics and other applied sciences. In the light of this fact, finding the analytical solutions of models expressed in terms of ordinary or partial differential equations has a great importance from the point of scientific progression. Therefore, new methods are always being developed. Integral methods are one of the most known methods [1]. The Laplace transform, the most well-known integral method, has been intensively used to solve the nonlinear and linear equations [1-3].

Many various methods such as double Laplace transform [4], double Sumudu transform [5], double Mellin transform [6], introduced by many researchers to find the solution of partial differential equations. The definition, properties and applications of the double Sumudu transform were described in [5]. In addition to these, triple Laplace transform was defined, related theorems and properties were presented in [7]. Thakur and Panda [8] had gave various properties of the triple Laplace transform.

The aim of this study, we believe for the first time, is to extend the Sumudu transform to triple Sumudu transform. Therefore, firstly, the triple Sumudu transform is defined and then various useful properties of the triple Sumudu transform are given. We believe that the triple Sumudu transform will be very useful in the solution of many partial differential equations and this relatively new transform will be used as a very effective tool in simplifying the calculations in many fields of engineering, mathematics and other applied sciences. In order to illustrate the ability...
of the method, Volterra integro-partial differential equation [9] solved by the triple Sumudu transform. On the other hand, in addition to these, this study is a part of the M.Sc. Thesis of the first author [10].

**Keywords** Sumudu Transform · Triple Sumudu Transform · Volterra integro-partial differential equations

**References**


UNDERSTANDING THE FAILURE IN DIFFERENTIAL AND INTEGRAL CALCULUS IN THE DEGREES OF ENGINEERING AT A HIGHER EDUCATION SCHOOL IN PORTUGAL

Maria Emília Bigotte de ALMEIDA¹, Araceli QUEIRUGA-DIOS², María José CÁCERES ²

¹Coimbra Institute of Engineering, Polytechnic Institute of Coimbra, Coimbra, Portugal
²University of Salamanca, Salamanca, Spain

Corresponding Author’s E-mail: ebigotte@isec.pt

ABSTRACT

Major difficulties are often found among students of engineering degrees in the Curricular Units (CU) of Mathematical Science area, particularly those related to Differential and Integral Calculus (DIC). The failure and drop out rates in CU-DIC have evidenced the need to question what methodologies and teaching approaches are applied and which assessment practices are related to their school success and lead to significant learning [1, 2, 3, 4]. A research work was developed with the objective of finding the reasons that lead the students to fail in the CU-DIC, taught in the 1st year of the engineering undergraduate degrees, in the Polytechnic Institute of Coimbra (ISEC). Applying a case study methodology, this paper presents a current diagnosis of CU-DIC in the ISEC, with the objective of defining a set of actions that will guide a research on the causes of failure in mathematics in engineering degrees. We intend to establish relationships between teaching methods and how students learn, and besides, build learning environments that lead to higher success with the co-responsibility of all actors in the educational process. The analysis of collected data allows us to conclude that the CU-DIC in ISEC maintains an identical distribution in the hourly load in several engineering degrees, contents are adjusted to each context taking into account CU of each degree. The data analysis found better results in the process that includes two examination moments without no relationship between class attendance, dropout and pass rates. The low participation of the students in the curricular assessment process, together with the pass rate obtained in the distributed evaluation, can lead us to question which set of basic and elementary level knowledge students need to master upon entering higher
education. This allow us the definition of a structured intervention in overcoming gaps.

**Keywords** Differential and Integral Calculus · Mathematical Knowledge in Engineering · Teaching and Learning

**References**


ANALYTICAL INVESTIGATION OF A TWO-MASS SYSTEM CONNECTED WITH LINEAR AND NONLINEAR STIFFNESSES

Md. Alal HOSEN

Department of Mathematics, Rajshahi University of Engineering and Technology, Rajshahi-6204, Bangladesh

Corresponding Author's E-mail: alal_ruet@yahoo.com

ABSTRACT

An iteration procedure has been developed based on the Mickens iteration method. This procedure also offers the angular frequencies and corresponding periodic solutions to the nonlinear vibration of a two-mass system connected with linear and nonlinear stiffnesses. A real-world case of this system is analysed and introduced. In this paper, the truncated terms of the Fourier series have been used and utilized in every step of iterations. The approximated results are compared with existing and corresponding numerical (considered to be exact) results. The obtained results are valid for whole ranges of vibration amplitude of the oscillations. The error analysis has carried out and shown acceptable results for the proposed iteration procedure. Effectiveness of the proposed iteration procedure found from comparison with other existing methods. The method is demonstrated by an example.

Keywords Nonlinear stiffnesses · Mickens iterative method · Two-mass system · Two-degree-of-freedom oscillation systems · Duffing equation

References


ON APPROXIMATION PROPERTIES OF SZÁSZ-MIRAKYAN OPERATORS

Melek SOFYALIOĞLU\textsuperscript{1}, Kadir KANAT\textsuperscript{2}

\textsuperscript{1,2}Ankara Hacı Bayram Veli University, Polatlı Faculty of Science and Arts, Mathematics, Ankara, Turkey

Corresponding Author’s E-mail: melek sofalioglu@hbv.edu.tr (https://orcid.org/0000-0001-7837-2785)

ABSTRACT

At this talk, we initially give a brief historical background of the Szász-Mirakyan operators. Then we construct a generalization of the Szász-Mirakyan operators and investigate the approximation properties of these operators. Moreover, we study uniform convergence and give quantitative estimation for the newly constructed operators. In order to show the rate of convergence, we use modulus of continuity.

Keywords Szász-Mirakyan operators · Linear positive operators · Modulus of continuity

References

CARDINALITIES AND RANKS OF NILPOTENT SUBSEMIGROUPS OF $C_n$

Melek YAĞCI

Department of Mathematics, Çukurova University, Adana, 01330, Turkey
Corresponding Author’s E-mail: msenol@cu.edu.tr

ABSTRACT

For $n \in \mathbb{N}$, let $T_n$ be the (full) transformation semigroup (under composition) on $X_n = \{1, \ldots, n\}$ under its natural order. A transformation $\alpha \in T_n$ is called order-preserving if $x \leq y$ implies $x\alpha \leq y\alpha$ for all $x, y \in X_n$ and decreasing (increasing) if $x\alpha \leq x$ ($x\alpha \geq x$) for all $x \in X_n$. The subsemigroup of all order-preserving transformations in $T_n$ is denoted by $O_n$, and the subsemigroup of all order-preserving and decreasing (increasing) transformations in $T_n$ is denoted by $C_n$ ($C_n^+$). It is a well-known fact [5, Corollary 2.7] that $C_n$ and $C_n^+$ are isomorphic semigroups.

Let $S$ be a semigroup, and let $A$ be any non-empty subset of $S$. Then the subsemigroup generated by $A$ is denoted by $\langle A \rangle$. The rank of a finitely generated semigroup $S$ is defined by

$$\text{rank}(S) = \min\{ |A| : \langle A \rangle = S \}.$$

An element $a$ of a semigroup $S$ is called indecomposable if $a \neq xy$ for all $x, y \in S$ that is, $a \in S \setminus S^2$. It is clear that every generating set of $S$ must contain all indecomposable elements of $S$. Thus, if $S = \langle A \rangle$ and $A$ consists entirely of indecomposable elements of $S$, then it is clear that $A$ is the minimum generating set of $S$.

The image, kernel and the fix of any transformation $\alpha \in T_n$ are defined by

$$\text{im}(\alpha) = \{ x\alpha : x \in X_n \},$$
$$\ker(\alpha) = \{(x, y) : x\alpha = y\alpha \text{ for all } x, y \in X_n \}\text{ and}$$
$$\text{fix}(\alpha) = \{ x \in X_n : x\alpha = x \},$$

respectively. An element $e$ of a semigroup $S$ is called idempotent if $e^2 = e$, and the set of all idempotents in $S$ is denoted by $E(S)$. It is literally immediate that for $\alpha \in T_n$, $\alpha$ is an idempotent if and only if $x\alpha = x$ for all $x \in \text{im}(\alpha)$. Equivalently, $\alpha$ is an idempotent if and only if $\text{fix}(\alpha) = \text{im}(\alpha)$. Moreover, an element $a$ of a semigroup $S$ with zero, denoted by $0$, is called a nilpotent element if $a^m = 0$ for some positive integer $m$. The set of all nilpotent elements of $S$ is denoted by $N(S)$. A semigroup $S$ with $0$ is called nilpotent if there exists $m \in \mathbb{N}$ such that $S^m = 0$. 


As in other algebraic theories it is an important problem to investigate the structure of nilpotent elements and nilpotent subsemigroups of semigroups. In this paper, we determine the structure of all the nilpotent subsemigroups of $C_n$.

**Keywords** Order-preserving/decreasing transformation · Rank · Nilpotent subsemigroup

**References**


THE NUMBER OF \( m \)-potent Elements in \( C_{n,Y} \)

Melek YAĞCI

Department of Mathematics, Çukurova University, Adana, 01330, Turkey
Corresponding Author’s E-mail: msenol@cu.edu.tr

ABSTRACT

For \( n \in \mathbb{N} \), let \( T_n \) be the (full) transformations semigroup (under composition) on \( X_n = \{1, \ldots, n\} \) under its natural order. A transformation \( \alpha \in T_n \) is called order-preserving if \( x \leq y \) implies \( x\alpha \leq y\alpha \) for all \( x, y \in X_n \) and decreasing (increasing) if \( x\alpha \leq x \) \((x\alpha \geq x)\) for all \( x \in X_n \). The semigroup of all order-preserving transformations in \( T_n \) is denoted by \( O_n \) and the semigroup of all decreasing (increasing) transformations in \( O_n \) is denoted by \( C_n \) \((C_n^+)\). It is a well known fact from [5, Corollary 2.7] that \( C_n \) and \( C_n^+ \) are isomorphic semigroups. The problem of finding certain combinatorial properties of \( O_n \) and \( C_n \), has been an important research area in Semigroup Theory. The index and period of an element \( a \) of a finite semigroup are the smallest values of \( m \geq 1 \) and \( r \geq 1 \) such that \( a^{m+r} = a^m \). An element with index \( m \) and period 1 is called an \( m \)-potent element. In [2], the authors obtained a formulae for the number of \( m \)-potent and \((m, r)\)-potent elements in \( T_n \). An element \( a \) of a finite semigroup \( S \) with a zero, denoted by 0, is called nilpotent if \( a^m = 0 \) for some positive integer \( m \), and moreover, if \( a^{m-1} \neq 0 \) then \( a \) is called an \( m \)-nilpotent element of \( S \). The set of all nilpotent elements of \( S \) is denoted by \( N(S) \). For any \( \alpha \in C_n \), it is clear that \( \alpha \) is an \( m \)-potent element for some \( m \in \mathbb{Z}^+ \).

Recall that \( n \)th Catalan number \( C_n \) is defined by

\[
C_0 = 1 \\
C_n = \frac{1}{n+1} \binom{2n}{n} = \frac{1}{n} \binom{2n}{n-1} 
\text{for } n \geq 1,
\]

(see for example p38 of [3]). From [4, Theorem 2.1 and Proposition 2.3] we know that

\[
|C_n| = |C_n^+| = C_n, \\
|N(C_n)| = |N(C_n^+)| = C_{n-1}.
\]

In general a transformation \( \alpha \in T_n \) is represented by the following tabular form:

\[
\alpha = \begin{pmatrix}
1 & \cdots & n \\
1\alpha & \cdots & n\alpha
\end{pmatrix}.
\]
For any $\alpha \in \mathcal{T}_n$ and for any non-empty subset $Y$ of $X_n$ we define

$$Fix(\alpha) = \{ x \in X_n : x\alpha = x \} \text{ and}$$

$$C_{n,Y} = \{ \alpha \in C_n : Fix(\alpha) = Y \},$$

the set $C_{n,Y}$ is first defined in [1].

The number of nilpotent elements in $C_n$ have been computed by Laradji and Umar in [4]. The number of $m$-potent and $(m, r)$-potent elements in $\mathcal{T}_n$ have been computed by Ayık, Ayık, Ünlü and Howie in [2]. However we have not seen any information about the number of $m$-potent elements in $C_{n,Y}$ in the literature. The aim of this paper is to find a formulae for any subset $Y$ of $X_n$, we obtain a formulae for the number of $m$-potent elements of $C_{n,Y} = \{ \alpha \in C_n : Fix(\alpha) = Y \}$. 

**Keywords** Order-preserving/decreasing transformation · $m$-nilpotent · $m$-potent

**References**


MODELLING TAP WATER CONSUMER RATIO

Meltem EKİZ¹, Osman Ufuk EKİZ²

¹,²Department of Statistics, Faculty of Science, Gazi University, Ankara, Turkey
Corresponding Author’s E-mail: ozmeltem@gazi.edu.tr

ABSTRACT

Unplanned population is known as a factor that causes water depletion in the world basins. Therefore, it is important to accurately predict the future ratios of tap water consumers using the same watershed to the population living in the specified area, in order to produce better water policies and take the necessary measures. Turkey, a country having land both in Asia and Europe and approximately living eighty-two million people, is exposed to dry seasons. European Environment Agency reported that Turkey will encounter moderate and high level water scarcity in many areas [1]. Thus, it is obvious that Turkey is a candidate country to experience problems on water scarceness. Considering that the population is predicted to be close to 100 million at 2030, [2], it is crucial to take precautions to avoid water shortages and to produce better water policies. Hence, it is important for a country to both follow and also predict the tap water consumer rates (TWCR) particularly to take measures on decreasing the negative effects of reduction in tap water that could occur in the very near future. TWCR are defined as the ratios of number of people using tap water to the number of population for each 26 grouped cities and are obtained for chosen years between 2001 and 2016. This study attempted to model tap water consumer ratios (TWCR) of watersheds in Turkey with growth curve models (GCMs). GCMs are preferred to model growth problems and are used to make short-term predictions [3, 4]. However, presence of outliers has an impact on parameter estimations and future predictions. Therefore, robust least median square (LMS) and M estimators are proposed to construct the GCMs. It is observed that the use of robust M estimator, which is resistant to outliers, gives more successful results. The predictions with the estimated third order GCM based on M estimator for 2020 and 2021 are founded to be approximately at the rate of 90 and 95, respectively.

Keywords Growth curve model · Outlier · Prediction · Robust estimators

References


COMPARISON OF NONTRADITIONAL OPTIMIZATION TECHNIQUES IN OPTIMIZATION OF SHELL AND TUBE HEAT EXCHANGER

Mert Akin INSEL¹, Inci ALBAYRAK², Hale Gonce KOCKEN²

¹Yıldız Technical University, Chemical Engineering Department, Turkey
²Yıldız Technical University, Mathematical Engineering Department, Turkey

Corresponding Author’s E-mail: makinsel@yildiz.edu.tr

ABSTRACT

Heat exchangers, which are the systems that allow heat transfer between two or more fluids, are widely used in industry, and they are an indispensable part of chemical processes. Thus, a considerable amount of research has been conducted in the optimization of these systems, especially regarding cost optimization. There are two main parameters affecting the cost of a heat exchanger. One is the surface area of the exchanger, which mainly affects the capital investment cost. The other one is the pressure drop, which mainly affects the operating cost. These parameters are interlinked; however, since the change in the surface area may result in a change in the pressure drop and vice versa due to the physical laws. In the case which is analyzed in this study, there are four main variables for the heat exchanger, which will be adjusted in order to obtain the optimum values for surface area and pressure drop: the number of tube side passages, shell inside diameter, baffles spacing, and tube outside diameter. In this study, we compare the success of the nontraditional optimization techniques such as generic algorithms, particle swarm optimization, artificial bee colony, and biography-based optimization for a shell and tube heat exchanger. In the considered three processes in which these optimization methods are utilized, we see that biography-based optimization has given the minimum cost for each process.

Keywords Optimization · Heat Exchanger · Cost Minimization

References


Fuzzy Modelling on Control of Heat Exchangers

Mert Akin INSEL¹, Inci ALBAYRAK², Hale Gonce KOCKEN²

¹Yıldız Technical University, Chemical Engineering Department, Turkey
²Yıldız Technical University, Mathematical Engineering Department, Turkey

Corresponding Author’s E-mail: makinsel@yildiz.edu.tr

Abstract

Heat exchangers are an indispensable part of many chemical processes. These are the systems that allow the transfer of heat between two or more fluids. The dynamics of heat exchangers are highly nonlinear; thus, many methods have been used to control these systems. Feedback, feedforward and cascade control loops can be presented as traditional methods. The type of controller used in these systems is also crucial, and controllers such as P, PI, PID are widely used in industry. Because of the complexity of the control, intelligent controllers like fuzzy logic controllers (FLCs) are also considered to provide an alternative to existing controllers, since FLCs provide ability to implement human knowledge and experience. In this study, control of a heat exchanger with process gain \( K_p = 50 \) and process time constant \( T_p = 30 \) was investigated by using a fuzzy-PID controller with feedback control loop. By comparing the conventional membership function for the fuzzy variables with proposed membership function, the response of the heat exchanger system to a step change of 10 units applied to the setpoint was interpreted.

Keywords Control · Heat Exchanger · Fuzzy Control

References


ON THE NEW FAMILIES OF GAUSS $k$-LUCAS NUMBERS AND THEIR POLYNOMIALS

Merve TAŞTAN$^1$, Engin ÖZKAN$^2$

$^1$Graduate School of Natural and Applied Sciences, Erzincan Binali Yıldırım University, Erzincan, Turkey

$^2$Department of Mathematics, Erzincan Binali Yıldırım University, Faculty of Arts and Sciences, Erzincan, Turkey

Corresponding Author’s E-mail: mervetaston24@gmail.com

ABSTRACT

In [1], Mikkawy and Sogabe gave a new family of $k$-Fibonacci numbers. In [2], Özkan et. all. defined a new family of $k$-Lucas numbers and give some properties about the family of these numbers. There are some works on polynomials of the families of $k$-Fibonacci numbers and $k$-Lucas numbers [3, 4]. In [5], Falcon and Plaza defined general $k$-Fibonacci numbers and showed that properties of these numbers were related with elementary matrix algebra. Finally [6], Taş presented $k$-Gauss Fibonacci numbers and give some properties related to these numbers. In this paper, we define the new families of Gauss $k$-Lucas numbers. We give the relationships between the family of the Gauss $k$-Lucas numbers and the known Gauss Lucas numbers. We also define the generalized polynomials for these numbers. We also obtain some interesting properties of the polynomials. We also give the relationships between the generalized Gauss $k$-Lucas polynomials and the known Gauss Lucas polynomials. Furthermore, we find the new generalizations of these families and the polynomials in matrix representation. Then we prove Cassini’s Identities for the families and their polynomials.

Keywords Lucas Numbers · Lucas polynomials · Gauss Lucas Numbers · Gauss Lucas polynomials · Cassini’s Identity

References


ON SPACES DERIVABLE FROM A SOLID SEQUENCE SPACE AND A NON–NEGATIVE LOWER TRIANGULAR MATRIX

Merve ULUDAĞ

Department of Mathematics, Faculty of Science, Çankırı Karatekin University, Tr-18100, Çankırı, Turkey
Corresponding Author’s E-mail: mmerveuludag@gmail.com

ABSTRACT

In this talk, we will give some recent results on the sequence of solid sequence spaces, each term of it generated by an infinite lower triangular matrix obtained by Johnson and Polat [1]. The scalar field will be either the real or complex numbers. Suppose that λ is a solid sequence space over the scalar field and A is an infinite lower triangular matrix with non-negative entries and positive entries on the main diagonal such that each of its columns is in λ. For each positive integer k, the k\textsuperscript{th} predecessor of λ with respect to A is the solid vector space of scalar sequences x such that A\textsuperscript{k}|x| is an element of λ. We denote this space by Λ\textsubscript{k} and λ itself will be denoted by Λ\textsubscript{0}. Under reasonable assumptions, these spaces inherit some topological properties from λ. We are interested in a projective limit of the infinite product of the Λ\textsubscript{k} consisting of sequences of sequences (x\textsuperscript{(k)}) satisfying Ax\textsuperscript{(k)} = x\textsuperscript{(k-1)} for each k > 0. We show that for interesting classes of situations including the cases when λ = l\textsubscript{p} for some p > 1 and A is the Cesàro matrix, the space of our interest can be non-trivial.

Keywords Solid sequence space · Infinite lower triangular matrix · Projective limit

References

NUMERICAL ANALYSIS OF AN ADAPTIVE NON-LINEAR FILTER BASED TIME REGULARIZATION MODEL FOR THE INCOMPRESSIBLE NON-ISOTHERMAL FLUID FLOWS

Mine AKBAŞ¹, Buse İNGENÇ²

¹Department of Mathematics, Düzce University, 81620, Düzce, Turkey
²Graduate School of Natural and Applied Sciences, Düzce University, 81620, Düzce, Turkey

Corresponding Author’s E-mail: mineakbas@duzce.edu.tr

ABSTRACT

This report proposes a modular filter based regularization model for the incompressible non-isothermal fluid flows, and a numerical method for solving that. The proposed model plugs a time relaxation term into Leray model which uses a deconvolution based indicator function. The task of this relaxation term is to drive the small scales to zero exponentially. The paper provides a complete finite element analysis of the model consisting of the stability and convergence analysis. Numerical experiments verify theoretical convergence rates and compare the approximate solutions of the model on a benchmark problem with the non-regularized finite element discretization of the flow problem.

Keywords  Time relaxation · Finite element method · Non-linear filtering

References

Approximate and Analytical Solutions for Nonlinear Fractional Systems

Muammer Ayata¹, Ozan ÖzkAN¹

¹Department of Mathematics, Faculty of Science, Selcuk University, Konya, 42003, Turkey
Corresponding Author’s E-mail: muammerayata55@gmail.com

Abstract

It is the first time in this study that we apply newly presented conformable Laplace decomposition method (CLDM) to fractional Drinfeld-Sokolov-Wilson equation (DSWE) and coupled viscous Burgers’ equation (CVBE) systems. While Drinfeld-Sokolov-Wilson equation system is considered as a mathematical model for shallow water waves, coupled viscous Burgers’ equation system is considered as a mathematical model for hydrodynamic turbulence. The results obtained in the applications are compared to exact solutions by the help of 2 and 3 dimensional figures and a table. It is seen that, CLDM is easy to use and it gives quite good results in even 2 steps. Also it can be said that the method will give researchers a new perspective for finding approximate and analytical solutions of many mathematical and physical problems.

Keywords Conformable fractional derivative · Drinfeld-Sokolov-Wilson system · coupled viscous Burgers’ equation · Shock wave theory · Adomian Decomposition Method

References


RUNGE-KUTTA-NYSTROM METHOD BASED ON ARITHMETIC MEAN FOR SOLVING SPECIAL SECOND ORDER ORDINARY DIFFERENTIAL EQUATIONS

Mukaddes ÖKTEM TURACI

Department of Computer Programming, Yenice Vocational School, Karabük University, 78700, Karabük, Turkey
Corresponding Author’s E-mail: mukaddesoktenturaci@karabuk.edu.tr

ABSTRACT

The Runge-Kutta methods are widely used by the researchers for solving ordinary differential equations. Recently, there has been a great deal of interest in the Runge-Kutta methods based on various means formulation for the numerical solution of ordinary differential equations. In this study, inspired by the Runge-Kutta method based on arithmetic mean, a third-order three-stage explicit Runge-Kutta-Nyström method based on arithmetic mean is presented for solving directly special second-order initial value problem of the form $y'' = f(x, y), y(x_0) = y_0, y'(x_0) = y'_0$. The stability properties of the proposed method are discussed and numerical examples are given to show the efficiency of the proposed methods compared to the existing Runge-Kutta and Runge-Kutta-Nystrom methods. The numerical results indicate that the proposed method is more efficient in terms of accuracy compared to other well known methods in literature.

Keywords Runge-Kutta method · Runge-Kutta-Nyström method · Arithmetic mean · Stability

References


REFINEMENTS OF BULLEN-TYPE INEQUALITIES FOR $s-$CONVEX FUNCTIONS VIA RIEMANN-LIOUVILLE FRACTIONAL INTEGRALS

Musa ÇAKMAK

Hatay Mustafa Kemal University, Reyhanlı Vocational School of Social Sciences, Turkey
Corresponding Author’s E-mail: enkucukcakmak@gmail.com

ABSTRACT

In this paper, the author establishes a new identity for differentiable functions and obtain some new inequalities for $s-$Convex Functions via Riemann-Liouville Fractional Integrals, afterwords applies these inequalities to construct inequalities for special means of two positive numbers.

Keywords Bullen’s inequality · $s-$convexity · Riemann-Liouville Fractional Integrals

References


A generalization of Hermite-Hadamard, Bullen, and Simpson inequalities via $h-$convexity

Musa ÇAKMAK

Hatay Mustafa Kemal University, Reyhanlı Vocational School of Social Sciences, Turkey

Corresponding Author’s E-mail: enkucukcakmak@gmail.com

ABSTRACT

In this paper, the author established a new identity for differentiable functions, afterward, he obtained some new general inequalities for functions whose first derivatives in absolute value at certain powers are $h$-convex by using the identity. On the other hand, a general inequality is studied, which gives Hermite-Hadamard, Bullen, and Simpson inequalities. Also, he gave an application for special means for arbitrary positive numbers.

Keywords $h$-convex function · Bullen inequality · Simpson inequality

References

THE AGREEMENT OF THE CONVENTIONAL DEFECT CORRECTION METHOD AND THE NOVEL DEFECT CORRECTING EXTRAPOLATION TECHNIQUE FOR LINEAR EQUATIONS

Mustafa AGGUL

Hacettepe University, Ankara, Turkey
Corresponding Author’s E-mail: mustafaaggul@hacettepe.edu.tr

ABSTRACT

The defect correcting extrapolation technique has been developed for, and applied to Oseen and Navier-Stokes flows successfully. This model solves equations with varying artificial viscosities (AV) and then employs an extrapolation idea to recover a corrected solution of the target problem. This way, each AV solution can be solved fully parallel, and hence reduce computational time dramatically comparing the conventional defect correction methods, which relies on the previously found AV step each time for a corrected solution. On the other hand, these two methods agree both in terms of theory and computational results for linear equations. In this talk, the narrator will address how these models coincide and exemplify their similarities with some linear equations.

Keywords  Defect correction · Defect correcting extrapolation · Artificial viscosity · Artificial diffusivity

References


ON HELICOIDAL HYPERSURFACES IN EUCLIDEAN 4-SPACE

Mustafa ALTIN¹, Ahmet KAZAN²

¹Technical Sciences Vocational School, Bingöl University, Bingöl, Turkey
²Department of Computer Technologies, Doğanşehir Vahap Kütük Vocational School, Malatya Turgut Özal University, Malatya, Turkey

Corresponding Author’s E-mail: maltin@bingol.edu.tr

ABSTRACT

It is known that a rotation hypersurface is defined as a hypersurface rotating a curve around an axis. In this context, if \( \alpha : I \subset \mathbb{R} \rightarrow \pi \) is a curve in a plane \( \pi \) of \( E^4 \) and \( l \) is a straight line in \( E^4 \), then a rotation hypersurface is defined by a hypersurface rotating the profile curve \( \alpha \) around the axis \( l \). Furthermore, if the profile curve \( \alpha \) rotates around the axis \( l \) and it simultaneously displaces parallel lines orthogonal to the axis \( l \), then the obtained hypersurface is called helicoidal hypersurface with axis \( l \). With the aid of these definitions, the differential geometry of rotation (hyper)surfaces and helicoidal (hyper)surfaces in 3 or higher-dimensional Euclidean, Minkowskian, Galilean and pseudo-Galilean spaces have been studied by scientists. For instance, finite type surfaces of revolution in a Euclidean 3-space have been classified in [1] and some properties about surfaces of revolution in four dimensions have been given in [2]. The third Laplace-Beltrami operator and the Gauss map of the rotational hypersurface in Euclidean 4-space and the general rotation surfaces in Minkowski 4-space have been studied in [3] and [4], respectively. Also, Dini-type helicoidal hypersurface in \( E^4 \) and Dini-type helicoidal hypersurfaces with timelike axis in \( E^4_1 \) have been studied in [5] and [6], respectively. In the present study, we deal with general helicoidal hypersurfaces in Euclidean 4-space. Firstly, we obtain the Gaussian and mean curvatures of helicoidal hypersurfaces in \( E^4 \) and give some results about the minimality and flatness of these hypersurfaces. After that, we study on the Gauss map of helicoidal hypersurfaces by obtaining the Gaussian and mean curvatures of it after long calculations. Finally, we give some visualizations for projections of some special helicoidal hypersurfaces and the Gauss map by fixing one of the three parameters of a hypersurface into 3-space.

Keywords Helicoidal hypersurface · Gauss map · Mean curvature
References

PERIODIC POINT RESULTS ON QUASI METRIC SPACES

Mustafa ASLANTAŞ

Department of Mathematics, Science Faculty, Cankiri Karatekin University, Cankiri, Turkey
Corresponding Author's E-mail: maslantas@karatekin.edu.tr

ABSTRACT

Fixed point theory has been broadly advanced and applied to various problems during last century. The applications of fixed point theorems comprise differing disciplines of mathematics, statistics, engenering and mathematical economics. In 1922, Banach proved an important theorem which is known as Banach contraction principle considered the beginning of the fixed point theory on metric spaces. Because of this principle’s applicability, this result has been extended and generalized in many various ways. In this sense, Boyd and Wong have been obtained a generalization of Banach contraction principle via nonlinear functions. After that, Ciric introduced the concept of periodic point. Since each fixed point is a periodic point, but the converse is not true. Also, a periodic point is a natural generalization of fixed point. Hence, periodic point results have been obtained by many authors in the abstract spaces. On the other hand, it was defined a new concept so called quasi metric spaces by removing the symmetry condition on metric spaces. This new concept has attracted interest of many authors, since the results obtained on standart metric spaces is not clear on this concept. Recently, Al Homidan et al introduced a notation which is called Q-function on quasi metric spaces to generalize ω-distance and τ-function. Hence, there are many studies in the literature obtained via the Q-functions. In this paper, we investigate some periodic point results on quasi metric spaces via Q-function. Hence, we generalize and extend some fixed point results existing in the literature. Finally, we present illustrative example to support our results.

Keywords Fixed point · Q-function · Quasi metric spaces

References


SOME FIXED POINT RESULTS FOR MULTIVALUED MAPPINGS ON M-METRIC SPACES

Mustafa ASLANTAŞ¹, Uğur SADULLAH²

¹,²Department of Mathematics, Science Faculty, Cankiri Karatekin University, Cankiri, Turkey
Corresponding Author's E-mail: maslantas@karatekin.edu.tr

ABSTRACT

In 1922, Banach proved an important theorem which is known as Banach contraction principle on metric space. This principle has a great number of applications in fields such as mathematics, computer and economic. Thus, this result was extended in many various ways. In this sense, Nadler proved a fixed point theorem for closed and bounded multivalued mappings on metric spaces. Then, this result was generalized by many authors by using Pompei Hausdorff metric (MT). However, Feng and Liu obtained a fixed point result for closed multivalued mapping without Pompei Hausdorff metric. After that, taking into account MT functions, Klim and Wardowski extended a fixed point result of Feng and Liu. On the other hand, Mathews introduced the concept of partial metric spaces and proved a fundamental fixed point theorem on these spaces. After, many authors studied fixed point theory both single valued and multivalued mappings on partial metric spaces. Recently, Asadi et al. give a new definition of M-metric spaces to generalize the concept of partial metric spaces. Moreover, they proved the M-metric version of the Banach contraction principle. Then, Hakan et al. obtained fixed point results for multivalued mappings of Feng-Liu type on M-metric spaces. In this paper, we prove some fixed point results for multivalued mappings of Klim-Wardowski type on M-metric spaces. Hence, our result generalizes a number of fixed point result for multivalued mappings in the literature. Finally, we present illustrative example to support our results.

Keywords  Fixed point · Multivalued mappings · M-metric spaces

References


K-ORDER FIBONACCI QUATERNIONS

Mustafa AŞCI ¹, Süleyman AYDINYÜZ²

¹,²Pamukkale University Science and Arts Faculty Department of Mathematics, Kinikli, Denizli, Turkey

Corresponding Author’s E-mail: mustafa.asci@yahoo.com

ABSTRACT

In this paper we define and study another interesting generalization of Fibonacci quaternions is called \( k \)-order Fibonacci quaternions. Then we obtain for \( k = 2 \) Fibonacci quaternions, for \( k = 3 \) Tribonacci quaternions and for \( k = 4 \) Tetranacci quaternions. We give generating function, the summation formula and some properties about \( k \)-order Fibonacci quaternions. Also, we identify and prove the matrix representation for \( k \)-order Fibonacci quaternions. The \( Q_k \) matrix given for \( k \)-order Fibonacci numbers is defined for \( k \)-order Fibonacci quaternions and after the matrices with the \( k \)-order Fibonacci quaternions is obtained with help of auxiliary matrices, important relationships and identities were established.

Keywords Fibonacci Numbers · \( k \)-Order Fibonacci Numbers · Quaternions · Fibonacci Quaternions · \( k \)-order Fibonacci Quaternions · Matrix representations · Q-matrix

References

RIGID BODY MOTION OF AN ELASTIC RECTANGLE UNDER THE EFFECT OF SLIDING BOUNDARY CONDITION

Onur ŞAHİN

Department of Mathematics, Giresun University, Giresun, Turkey

Corresponding Author’s E-mail: onur.sahin@giresun.edu.tr

ABSTRACT

Dynamics of elastic structures are of interest for numerous studies in modern industrial applications, see [1]. One of the important points of this topic is developing the mathematical models which reveal the eigenfrequencies of the system from the traditional equations of rigid body dynamics, see [2]. However, the exact formulation of the eigenfrequencies and corresponding eigenforms of such problems are commonly given by lengthy transcendental relations. A considerable number of studies in recent years, therefore, have focused on developing perturbation approaches allowing further insight into the dynamic response of considered elastic structures, e.g. [3] and [4]. It is well known that the conventional equations of rigid body motion are an application of Newtonian mechanics to elastic solids. We mention [5] which suggests a new methodology leading a better approximation to Newton’s second law of motion for rigid macroscopic bodies. A distinguished elastodynamic homogenization theory for periodic and random inhomogeneous media was presented in [6]-[9], including effective constitutive relations which are non-local in space and time.

In this work, the in-plane dynamic problem for an elastic rectangle with the sliding boundary condition in case of low-frequency motion related to a typical time scale is considered. The governing equations and the boundary conditions are given in terms of the scaled variables and a perturbation scheme is derived by expanding the stresses and displacements in asymptotic series through the associated small parameter related to low frequency. As might be expected the leading order terms in these expansions correspond to rigid body motions. It should also be emphasized that the asymptotic study of boundary-value problems usually agrees with the Saint-Venant principle which states that statically self-equilibrated loads cause only local disturbances that do not propagate far away from a loaded area, see [2] and [10]. At the next order of the asymptotic expansion, we have a boundary value problem...
for a system of partial differential equations. The aim of further study is to find the solution to the next order problem and present the asymptotic solution of the elastic rectangle.

**Keywords** Low-frequency · Perturbation · Elastic rectangle · Rigid body motion

**References**


ABSTRACT

Prey-predator interaction has always been an important issue in mathematical modeling of ecological processes. Lotka-Volterra model is the simplest model of predator-prey interactions. The model was developed independently by Lotka (1925) and Volterra (1926) [1,2]. The Lotka-Volterra equations, also known as the predator-prey equations, are a pair of first-order nonlinear differential equations, frequently used to describe the dynamics of biological systems in which two species interact, one as a predator and the other as prey. The model of Lotka and Volterra is not very realistic. It does not consider any competition among prey or predators. As a result, prey population may grow infinitely without any resource limits. Predators have no saturation: their consumption rate is unlimited. The rate of prey consumption is proportional to prey density. Thus, it is not surprising that model behavior is unnatural showing no asymptotic stability. However, numerous modifications of this model exist which make it more realistic. One of these modifications is to introduce the Allee effect [3,4,5]. In 1931, an ecologist Warder Clyde Allee described that per capita growth rate of prey is negative or an increasing function at low population density. This type of mechanism is known as Allee effect. The Allee effect is a nonlinear phenomenon exhibited in the population dynamics of sparse populations in which the per capita population growth rate increases with increasing population density. In sufficiently sparse populations, the Allee effect may lead to extinction and is known to generate a threshold in the probability of establishment when presented as a function of introduced population size or density. Recently, many researchers have paid attention to study the dynamics of discrete time prey-predator models with Allee effect.

In this study, we consider discrete-time prey-predator model with Allee effect. We show existence of the fixed point of the presented discrete-time model. We analyze
the topological classifications of these fixed points of the model and investigate stability analyses of the all fixed points. We give numerical simulations to support obtained theoretical finding.

**Keywords**  Allee effect · Stability · Fixed Point

**References**

[1] Lotka, AJ., Elements of Mathematical Biology, Williams & Wilkins, Baltimore (1925).


The constant technological development has changed profoundly the way we interact with each other, think and work, so the skills required by employers for new professionals graduated from universities also changed. This new paradigm requires educators, especially higher education teachers, to improve teaching and learning methodologies in order to promote critical thinking for solutions to complex and decision-making problems. Education should also take into account communication and collaboration, as well as the ability to master new technologies, avoiding the risks of their misuse. Along with them, skills such as critical thinking, creativity, originality, initiative, persuasion and negotiation should also be considered in training and they are in line with European Union (2018) documents along with the need to support the development of key skills in the acquisition of science, technology, engineering and mathematics (STEM) skills, taking into account their links with the arts, creativity and innovation and motivating young people to engage in STEM/(STEAM) careers. According to the report of the World Economic Forum (2018), politicians, educators and workers’ representatives will gain if there is a deeper understanding of the labour market, as well as preparation for the ongoing changes. As teachers of higher education in science and technology and in particular in engineering, and as participants in the Erasmus+ CRITHINKEDU project (based on the knowledge and experience of European Higher Education Institutions, Enterprises and Non-Governmental Organisations), we believe that there should exist a constant concern to improve the quality of learning in higher education institutions in order to address the need for well-trained and well-educated citizens with skills tailored to labour market requirements. The present work arises from this concern in the Mathematics field and from the changes that the authors experienced fostered by the teacher-training course in Education for Critical Thinking of the Erasmus+ CRITHINKEDU project. This is a qualitative work, and we present a description of three different courses that were altered after the training course. The changes are presented at the level of planning, teaching and learning.
strategies used, as well as the learning assessment. Despite the introduced alterations, we know that we still have to continue the path started with training, as there are still aspects to improve to achieve all the aspects of the CRITHINKEDU project framework, in the teaching practices described, namely those that cover the critical thinking assessment.

**Keywords** Mathematics · Critical thinking · Teaching methodology

**References**


A STUDY ON THE TANGENT BUNDLE OF RIEMANNIAN MANIFOLD ENDOWED WITH THE BRONZE STRUCTURE

Rabia ÇAKAN AKPINAR

Kafkas University, Faculty of Arts and Sciences, Department of Mathematics, Turkey
Corresponding Author’s E-mail: rabiacakan@kafkas.edu.tr

ABSTRACT

In this work, the bronze Riemannian structures are determined. An integrability condition for bronze Riemannian structure is investigated via the Tachibana operator. An example for the tangent bundle of Riemannian manifold endowed with the Sasaki metric and the bronze structure is studied.

Keywords Bronze structure · Pure tensor · Riemannian manifold · Tangent bundle · Sasaki metric

References

ESTIMATION OF TURKEY AND WORLD POPULATION IN THE YEAR 2050 IN VIEW OF LOCAL AND NONLOCAL FRACTIONAL DERIVATIVES

Ramazan OZARSLAN

Department of Mathematics, Firat University, Elazig, Turkey
Corresponding Author’s E-mail: ozarslanramazan@gmail.com

ABSTRACT

In this work, the study aims to estimate Turkey and the world population more accurately in 2050 by using World Bank real population data. In view of local and non-local fractional derivatives, which are conformable, Caputo, Caputo Katugampola, population growth model is evaluated. The results are analyzed by using least squares method and compared to the World Bank’s estimates for 2050.

Keywords Population growth · Local derivative · Fractional calculus · Mathematical Modeling

References

THE EFFECT OF CAPUTO FRACTIONAL DERIVATIVE CONCERNING ANOTHER FUNCTION ON DEFLECTION OF HORIZONTAL BEAMS

Ramazan OZARSLAN

Department of Mathematics, Firat University, Elazig, Turkey
Corresponding Author’s E-mail: ozarslanramazan@gmail.com

ABSTRACT

This study investigates the effect of Caputo fractional derivative concerning another function on beam deflection under the vertical load for determining transverse deflection. The standard model includes a fourth order differential model and is analysed in this study in a broader way through the fractional case while the order between $3 < \alpha < 4$ and the fractional derivative concerning another functions like transcendental, rational functions, etc. analytically under various boundary conditions.

Keywords $\psi$—Caputo fractional derivative · Beams · Buckling load

References

ON LIGHTLIKE SUBMERSIONS

Ramazan SARI

Amasya University, Turkey
Corresponding Author’s E-mail: ramazan.sari@amasya.edu.tr

ABSTRACT

We introduce semi-slant lightlike submersions from indefinite Hermitian manifold onto a lightlike manifold. We obtain characterizations; investigate the geometry of foliations which arise from the definition of this new submersion. After we investigate the geometry of foliations, we obtain necessary and sufficient condition for base manifold to be a locally product manifold and proving new conditions to be totally geodesicness, respectively.

Keywords Lightlike submersions · Slant lightlike submersions · Lightlike manifolds

References

MMR ENCRYPTION ALGORITHM AS AN ALTERNATIVE ENCRYPTION ALGORITHM TO RSA ENCRYPTION ALGORITHM

Remzi AKTAY
Keçiören Şehit Halil İşlar Middle School, Mathematics Teacher, Turkey
Corresponding Author’s E-mail: aktayremzi@outlook.com

ABSTRACT

In this study we aimed to try to develop an alternative algorithm with the purpose of (key encryption algorithm) RSA’s current disadvantages and possible problems when we switch to the future quantum computers. Because of the usage method of the RSA, the values during the key making process are limited. Owing to this feature of this method, may side channel attacks have been made and some of them have had good results. Especially at the attack of Shor algorithm and Acoustic crypto analysis; Shamir and his team had good results in 2013. We also know at the error analysis attack and Bellcore attack in 2010. Firstly in the algorithm that has planned to be composed; the keys will take infinite values. And also besides the closed keys there will be observational keys which are worked only in these algorithms. In the event of the closed keys are found; these observational keys will step in.

While composing this algorithm at first, an equation has been made. This equation is $a^{10(n-1)} - b^{10(n-1)} \equiv 0 (mod 10^n)$. In this equation we started to the algorithm via making a and b as twin prime numbers. With this equation we composed the closed keys by means of modular arithmetic rules and Euclid Algorithm. And we make a working algorithm principle using Euler Theory, Euler Function and factorizing.

The first difference of this algorithm from the open keys algorithm is the observational keys. In the event that the open keys are stolen or broken, without the observational the keys data and secret text cannot be reached. The second difference is that the open key, closed key, and the observational key can have an infinite value. The third difference is this algorithm has its own character code chart. And also this algorithm is safer against the side channel attacks. While the algorithm’s control key changes, the closed and open key of the message sender also changes. Besides, the twin prime numbers of the equation can be changed to make an update. This algorithm can be used for secure e-mail, e-commerce, mobile banking and secure data transfer.
**Keywords** Encryption · closed key · Observation key · Euclid Algorithm · Euler’s Theorem · Side Channel Attacks · Open Key encryption

**References**


RESULTS ON SET-VALUED PRESÍČ TYPE MAPPINGS

Seher Sultan YEŞİLKAYA¹, Cafer AYDIN²

¹Institute of Science and Technology, Kahramanmaraş Sütçü İmam University, Kahramanmaraş, 46040, Turkey
²Department of Mathematics, Kahramanmaraş Sütçü İmam University, Kahramanmaraş, 46040, Turkey

Corresponding Author’s E-mail: sultanseher20@gmail.com

ABSTRACT

Fixed point theory concern itself with a very basic mathematical setting. It is also well known that one of the fundamental and most useful results in fixed point theory is Banach fixed point theorem. In 1922 Banach [1], introduced the famous fixed point theorem, also known as the Banach contraction principle. Throughout the years, several extensions and generalizations of this principle have appeared in the literature. Furthermore, among the different generalization of Banach contraction principle, Prešić [2] in 1965 gave a contractive condition on finite product of metric spaces and proved a fixed point theorem. Also Ćirić and Prešić [3], Abbas et al. [4], Shukla and Sen [5] and Yeşilkaya et al.[6] extended and generalized these results. On the other hand, there are several applications of Prešić type contractions, e.g., in the convergence of sequences in solving the nonlinear difference equations and the nonlinear inclusion problems in the convergence problems of nonlinear matrix difference equations etc.. Set-valued analysis is an important extension of the general concepts studied in mathematical analysis. Fixed point theory for set-valued operators is an important topic of set-valued analysis. Nadler [7] extended the Banach contraction principle to set-valued mappings by using the Hausdorff metric. Inspired by the results of Nadler, the fixed point theory of set-valued contraction using this Hausdorff metric has been further developed in different directions by many authors. Recently, Wardowski [8] has introduced a new concept of contraction, and proved a fixed point theorem which generalizes the Banach contraction principle in a different direction than in the known results from the literature in complete metric spaces.

In this work, we introduce a new concept of contraction called set-valued Prešić type \( F \)-contraction mapping on metric spaces and prove some fixed point results for a mapping \( M : X^r \to CB(X) \). These results extend the main results of many comparable results from the current literature. Also, we provide example showing that our main theorem are applicable.

Keywords Fixed point · metric spaces · Prešić type contraction
References


NEW RESULTS ON EXPANSIVE MAPPINGS

Seher Sultan YEŞİLKAYA¹, Cafer AYDIN²

¹Institute of Science and Technology, Kahramanmaraş Sütçü İmam University, Kahramanmaraş, 46040, Turkey
²Department of Mathematics, Kahramanmaraş Sütçü İmam University, Kahramanmaraş, 46040, Turkey

Corresponding Author’s E-mail: sultanseher20@gmail.com

ABSTRACT

The study of expansive mappings is a very interesting research area in fixed point theory. Wang, et al.[1] proved some fixed point theorems for expansion mappings, which correspond to some contractive mappings in metric spaces. Rhoades [2] and Taniguchi [3] generalized the results of Wang for pair of mappings. Thereafter, several authors obtained many fixed point theorems for expansive mappings. For more details see [4, 5]. Fixed point theory in metric spaces is an important branch of nonlinear analysis, which is closely related to the existence and uniqueness of solutions of differential and integral equations. There are many generalizations of the concept of metric spaces in the literature. Ran and Reurings [6] proved a fixed point theorem on a partially ordered metric space. Then several authors considered the problem of existence (and uniqueness) of a fixed point for contraction type operators on partially ordered sets. Recently, Jleli and Samet [7] introduced a new type of contraction which is called the $\theta$-contraction and established some new fixed point theorems for such a contraction in the context of generalized metric spaces.

The purpose of this study is to present some fixed point results for self-generalized contractions in partially ordered metric spaces. Also, we introduce a new approach to expansion mappings in fixed point theory. We introduce the concept of $\theta$-expansive mappings in metric spaces and prove fixed point theorems for such mappings. Following, we provide examples to show the significance of such mappings. Secondly, we give some fixed point results for $\theta$-expansive mappings especially on a partially ordered metric spaces. These results extend the main results of many comparable results from the current literature. Finally, some examples are present to support the new theorems and results. Also, these examples show the new class of $\theta$-expanding mapping is not included in expanding mapping known in literature.

Keywords Fixed point · Metric spaces · Expansion mappings
References


ON A TYPE-2 FUZZY APPROACH TO SOLUTION OF SECOND ORDER INITIAL VALUE PROBLEM

Selami BAYEĞ¹, Raziye MERT¹, Tahir KHANIYEV², Ömer AKIN²

¹Turkish Aeronautical Association University
²TOBB University of Economics and Technology

Corresponding Author’s E-mail: sbayeg@thk.edu.tr

ABSTRACT

Type-2 fuzzy sets (T2FS) were firstly introduced by Zadeh in 1975 as a natural extension of fuzzy sets [1]. A T2FS is defined by a fuzzy membership function, the membership of which is a fuzzy set in the unit interval [0, 1] rather than a point in [0, 1]. Hence type-2 fuzzy sets are helpful in models where the exact form of membership functions cannot be determined or where membership functions are themselves imprecise in nature [2]. Practically, type-2 fuzzy sets are complex by nature and their logic is characterised by expensive computational cost. Many representations are developed to overcome this obstacle, and still the applicability of type-2 fuzzy sets is very limited in applications. A differential equation with type-2 fuzzy initial values is called a type-2 fuzzy initial value problem (T2FIVP). Although there are many studies on fuzzy initial value problems, there are only a few studies on type-2 fuzzy initial value problems [3, 4, 5].

In our work we have developed an algorithm to find the solutions of the second order linear type-2 fuzzy initial value problems by using the (α, β)-cut representation of type-2 fuzzy numbers (T2FNs) given in [6]. These type-2 fuzzy solutions are firstly constructed from the solution of the crisp problem using the Zadeh’s extension principle. Then type-2 fuzzy solutions are obtained using the (α, β)-cut representations of T2FNs. In this representation, type-2 fuzzy solutions are represented by lower and upper solutions such that each of these solutions has another lower and upper parts corresponding to (α, β)-cut solutions. To prevent the switching points of these lower and upper solutions we have used Heaviside step function. Since the second order linear differential equations have a variety of applications such as the motion of a particle, the vibration of a spring and the flow of electrons in an electric circuit, as an application of the method we have applied the method to mechanical vibrations and RLC-circuits as applications.
Keywords  Type-2 fuzzy sets · Second order type-2 fuzzy differential equations · Zadeh’s extension principle

References

EXISTENCE AND UNIQUENESS RESULTS FOR A NONLINEAR FRACTIONAL DIFFERENTIAL EQUATIONS OF ORDER $\sigma \in (1, 2)$

Sinan Serkan BİLGİÇİ\textsuperscript{1}, Müfit ŞAN\textsuperscript{2}

\textsuperscript{1,2}Department of Mathematics, Faculty of Science, Çankırı Karatekin University, Tr-18100, Çankırı, Turkey

Corresponding Author’s E-mail: sinan.serkan.bilgici@gmail.com

ABSTRACT

The importance of existence and uniqueness theorems in the study of initial value problems is unquestionable because, without them, one cannot understand modelled systems correctly and cannot make predictions about the behaviour of them. Recently, with the popularity of fractional derivative operators, the equations involving these operators have begun to be studied in detail (See, [1]-[3]). In this research, we investigate the existence and uniqueness of solutions of an initial value problem involving a nonlinear fractional differential equation of order $\sigma \in (1, 2)$ in the sense of Riemann-Liouville derivative. Unlike the previous studies (See,[2],[7]), initial values of the problem we considered will be taken as nonhomogenous. As shown in [4]-[6], an initial value problem including a nonlinear fractional differential equation of order $\sigma \in (0, 1)$ has no continuous solution when the problem has a nonhomogenous initial value and the right-hand side of the equation is continuous on $[0, T] \times \mathbb{R}$. Since the similar problem arises in our investigation, we will reveal some appropriate necessary and sufficient conditions under which the problem has at least one solution. Moreover, we will study on the uniqueness of the problem as well. The existence and uniqueness of solutions will be investigated in Banach space $C^\beta([0, T])$, where $0 < \beta < 1$ and $T > 0$, and it is endowed with the following norm

$$||\omega||_\beta = ||\omega||_\infty + ||D^\beta\omega||_\infty,$$

where $||.||_\infty$ stands for the supremum norm defined on the class of continuous functions and $D^\beta$ represents the Riemann-Liouville fractional derivative. Moreover, Schauder fixed point theorem and some well known theorems will be used to prove our claims.

Keywords  Fractional differential equations · Existence theorem · Riemann-Liouville derivative · Initial-value problem.
References


A major part of any training process in education is the assessment of the knowledge and skills achieved by students at a given stage. Traditional methods of teaching, learning and testing are still essentially related to the content of the relevant teaching material, according to university programs. In recent years, this model has been gradually replaced by many new methods, mostly computer-assisted. Requirements are formed for a greater connection of the studied material with its possible applications in practice, in the sense of the concept of competence. The aim of this work is to evaluate the student’s performance in mathematics using a competency-based assessment approach. The assessment methodology applies eight well-specified mathematical competencies, developed in the literature, intended for engineering higher education [1,2]. In addition, some new assessment tools obtained as a result of the RULES_MATH project, recently published in [3-5], are applied. This paper presents an example assessment test, which combines questions on elements of mathematical analysis, numerical methods and applied statistics, as well as an additional small practical project. The practical project requires students to apply a set of mathematical knowledge to formulate problems, solve them with or without specialized software, and interpret the resulting solutions. The developed assessment test was used as a final exam in Applied mathematics with a course of first-year students in the specialty of Business Information Technology at Plovdiv University Paisii Hilendarski, Bulgaria. The accumulated empirical data are processed by using powerful machine learning techniques, such as Classification and Regression Trees (CART), capable to identify the influence of different factors and their importance on the acquired level of knowledge and competencies of the students. The test results show that the mastery of the content-based material during the school year and the level of the mathematical competence, measured by the outcome of the small practical project, differ in favor of the formal training. This requires a
change in teaching methods and strengthening the competency-oriented component to increase the overall level of competencies in mathematics of students.

**Keywords** Mathematical competence · Assessment test · Student performance

**References**


GENERALIZED K-ORDER FIBONACCI HYBRID AND LUCAS HYBRID NUMBERS

Süleyman AYDINYÜZ¹, Mustafa AŞCI²

¹,²Pamukkale University Science and Arts Faculty Department of Mathematics, Kinikli, Denizli, Turkey
Corresponding Author’s E-mail: aydinyuzyeleyman@gmail.com

ABSTRACT

In this study, we define the generalized $k-$order Fibonacci and Lucas numbers. We also give some important result about the generalized $k-$order Fibonacci and Lucas numbers with special choices. The aim of this study is to introduce a new sequence of numbers called the generalized $k-$order Fibonacci Hybrid and Lucas Hybrid numbers. Then we obtain for $k = 2$ the Horadam Hybrid numbers and with the special choices usual Fibonacci, Lucas, Pell, Pell-Lucas, Jacobsthal and Jacobsthal-Lucas Hybrid numbers. For $k = 3$ and some special choices we get the Tribonacci Hybrid numbers. We give the generating functions and some properties about the generalized $k-$order Fibonacci Hybrid and Lucas Hybrid numbers. Also, we identify and prove the matrix representation for the generalized $k-$order Fibonacci Hybrid and Lucas Hybrid numbers. The $Q_k$ matrix given for $k-$order Fibonacci numbers is defined. Important relationships and identities are established.

Keywords Fibonacci and Lucas numbers · $k$-order Fibonacci and Lucas Numbers · Generalized $k$-order Fibonacci and Lucas numbers · Hybrid numbers · Generalized $k$-order Fibonacci Hybrid and Lucas Hybrid numbers · Matrix representations · $Q$-matrix

References

K-ORDER GAUSSIAN FIBONACCI POLYNOMIALS AND APPLICATIONS TO THE CODING/DECODING THEORY

Süleyman AYDINYÜZ 1, Mustafa AŞCI 2

1,2 Pamukkale University Science and Arts Faculty Department of Mathematics, Kinikli, Denizli, Turkey
Corresponding Author’s E-mail: aydinyuzsuleyman@gmail.com

ABSTRACT

In this paper we define k-order Gaussian Fibonacci polynomials with boundary conditions and give the generating function, explicit formula and some identities for k-order Gaussian Fibonacci polynomials. We introduce the matrix representation and we obtain the k-order Gaussian Fibonacci Polynomials matrix. We define a new coding theory called k-order Gaussian Fibonacci Polynomials coding theory and establish the code elements for values of k. This coding/decoding method bound to the $Q_k(x)$, $R_k(x)$ and $E_{k,n}(x)$ matrices. So, this method is different from the classical algebraic coding. Consequently, with this method, we move the coding theory onto a complex space which is a different field. Therefore, new areas are created.

Keywords Fibonacci Numbers · Gaussian Fibonacci numbers · k-order Gaussian Fibonacci numbers · Fibonacci polynomials · k-order Gaussian Fibonacci Polynomials · Coding/decoding method · k-order Gaussian Fibonacci polynomial coding/decoding theory

References


FINITE DIFFERENCE METHOD FOR FRACTIONAL PARTIAL DIFFERENTIAL EQUATION DEFINED BY ATANGANA-BALEANU CAPUTO (ABC)

Sümeyye EKER¹, Mahmut MODANLI ²

¹,² Harran University, Turkey

Corresponding Author’s E-mail: sumeyyeeker34@gmail.com

ABSTRACT

In this paper fractional partial differential equation defined Atangana- Baleanu Caputo equation is viewed. The exact solution is calculated for partial differential equation defined by Atangana- Baleanu Caputo fractional derivative operator depend on initial boundary value problems. Laplace transformation method is used for the exact solution. Finite difference schemes are constructed and stability estimates are obtained for this equation. For this problem, the stability of difference schemes is shown. This technique has been applied at certain levels of the ABC partial partial derivative. Approximation solution confirm the accuracy and effectiveness of the technique.

Keywords Finite difference method · Fractional order Atangana-Baleanu Caputo partial differential equation · Stability

References


Graph theory are studied different areas such as mathematics, computer science, information and chemistry sciences. Furthermore, it becomes one of the most powerful mathematical tools in the analysis and study of network architecture. Networks are often represented by graphs, which are identified by their vertices and edges where vertices are nodes and edges are links. The stability and reliability of a network are of prime importance to network designers dealing with the nodes and links. The vulnerability value of a communication network shows the resistance of the network after the disruption of some centers or connection lines before a communication breakdown occurs. Several existing parameters have been proposed in the literature to measure network vulnerability, such as the domination number $\gamma(G)$, the average lower domination number $\gamma_{av}(G)$, the residual domination number $\gamma^R(G)$, the average lower residual domination number $\gamma^R_{av}(G)$, the link residual domination number $\gamma^{LR}(G)$ and the average lower link residual domination number $\gamma^{LR}_{av}(G)$.

Generalized caterpillar graphs are very important tree network topologies. In this study, the residual domination number, the average lower residual domination number, the link residual domination number and the average lower link residual domination number of two classes of generalized caterpillar graphs are obtained, and also these values are compared.

**Keywords** Graph vulnerability · Domination in graphs · Trees · Generalized caterpillar graphs

**References**


ABSTRACT

In this study, some special spacelike surfaces in de-Sitter 3-space are discussed. Spacelike ones of normal, binormal and Darboux surfaces were investigated in De-Sitter 3-space. Due to the variation in the causal character of a vector, a curve, and a surface in this space, this space has a rich structure. This rich structure gives the opportunity to diversify the surfaces we consider. The situation of normal, binormal and Darboux surfaces that are spacelike are constant angle surfaces were investigated. Since these surfaces are built on a curve, the causal character of the curve is also very important for these types of surfaces. The concept of constant angle surface, similar to the Helisoid surfaces in Lorentz space, which has many applications in the technique, occupies a huge place in the establishment of these spacelike surfaces in the de-Sitter space. Also, since these special surfaces will be constructed as constant angle surfaces, attention should be paid to the causal character of the constant vector area of the de-Sitter space and the causal character of the fixed angle when creating these surfaces. In this study, with all these important situations, using the rich structure of de-Sitter space, the spacelike ones of normal, binormal and Darboux surfaces will be introduced. In recent years, similar types of surface types well known in Euclidean space are also being investigated in Sitter space. De-Sitter space is a model for physical events and many physical events can be interpreted in this space. Since the surface types in these spaces will guide the areas related to our daily life, this type of surface is of great importance. It is possible to see this from the structures used in the history of architecture. The importance of such surfaces in this space is clear and this study will contribute to geometry.

Keywords Normal Surface · Binormal Surfaces · Darboux Surfaces
References


HAMILTON MATRICES OF REAL QUATERNIONS AND SPINORS

Tülay ERİŞİR¹, Emrah YILDIRIM²

¹,²Erzincan Binali Yıldırım University, Turkey
Corresponding Author’s E-mail: emrhyldrm@hotmail.com

ABSTRACT

The theory of spinors, especially used in applications to electron spin and theory of relativity in quantum mechanics, was expressed by B. L. van der Waerden in 1929. The introduction of spinors is one of the most difficult topics in quantum mechanics. Even if the spin-1/2 is considered, some fundamental aspects of spinors, such as the effects of rotation on spinors, turn out to be difficult to explain. Spinors appear to be closely related to the electromagnetic theory. According to physicists, spinors are multilinear transformations. Thanks to this feature, spinors are mathematical entities somewhat like tensors and allow a more general treatment of the notion of invariance under rotation and Lorentz boosts. For mathematicians, spinors are vectorial objects and their multilinear features do not play any role. Also, spinors have one-index. In discussing vectors and tensors there are two ways in which we can proceed: the geometrical and analytical. To use the geometrical approach, we describe each kind of quantity in terms of its magnitudes and directions. In the analytical treatment, we use components. Spinors were first studied by Cartan in a geometrical sense. Cartan was one of the founders of Lie group theory which is one of the most important topics of mathematics and has many physical applications. So, this study is a very impressive reference in terms of the geometry of the spinors since this gives the spinor representation of the basic geometric definitions [2]. In geometrical meaning, another study was made by Vivarelli. In this study, Vivarelli established a one-to-one linear relationship between the quaternions and spinors. In addition, using the relationship between the rotations in quaternions and three-dimensional Euclidean space, Vivarelli actually obtained the spinor representation of the rotations in Euclidean space [4]. In this paper, we have introduced spinors given by Cartan algebraically [1, 2]. Then, considering the Hamilton matrices of real quaternions we have given spinor representations of these Hamilton matrices. Moreover, we have obtained eigenvalues and eigenvectors of these spinor matrices. In addition that, we have proved some prop-
properties of spinor matrices. Finally, we have expressed some theorems and corollaries. So, we have brought a new perspective to real quaternions.

**Keywords** Spinors · Real quaternions · Hamilton matrices

**References**

Due to the Bologna Accord, the teaching of mathematics has undergone important changes. Some of the most visible modifications have been the need to complement the traditional teaching-learning process with practical, real-life cases and the possibility to reinforce the introduction and usage of key concepts through mathematical software. Nowadays, there exist many computational packages dealing with mathematics, some of the best-known being Mathematica and Matlab. However, although they are very complete and powerful, they demand the use of commercial licences, which can be a problem for some education institution or in the cases where students desire to use the software in an unlimited number of devices or to access from several of them simultaneously.

In this contribution, we show how to apply GeoGebra and WolframAlpha to the teaching of Calculus for first-year university students. While GeoGebra is an interactive geometry, algebra, statistics, and calculus application available both as an online resource and a native application in Windows, macOS, and Linux systems, WolframAlpha is a computational knowledge engine developed by a subsidiary of Wolfram Research, the company behind Mathematica. However, unlike that product, WolframAlpha can be accessed by any individual as a web service free of charge. One of the key aspects of WolframAlpha is the possibility to use natural language and Mathematica syntax for requesting computations, which allows users to benefit from a large amount of Mathematica resources.

Being able to use GeoGebra and WolframAlpha as web services without downloading and installing software is another important advantage, as it avoids the need to have administrator rights to use those computational engines, which typically represents a problem in education centres where lab computers are locked so students cannot inadvertently install malware that can compromise the university’s network.
As the best way to show a topic in mathematics is to provide usage examples, this contribution reviews the main topics associated to a first-year Calculus class (limits, continuity, derivatives, curve interpolation and integrals), providing the commands used in GeoGebra and WolframAlpha for the computations and concrete examples used in actual Calculus classes.

**Keywords** Calculus · Freeware · Mathematics · Software

**References**


CHARACTERIZATIONS AND INTEGRAL FORMULA FOR GENERALIZED RICCI ALMOST SOLITONS

Yasemin SOYLU

Giresun University, Department of Mathematics, 28100, Giresun, Turkey
Corresponding Author’s E-mail: yasemin.soylu@giresun.edu.tr

ABSTRACT

The concept of Ricci solitons was introduced by Hamilton [6] in 1988 that is natural generalizations of Einstein metrics. Ricci solitons also correspond to self-similar solutions of Hamilton’s Ricci flow [5] and often arise as limits of dilations of singularities in the Ricci flow. Ricci solitons are of interests to physicists as well and are called quasi-Einstein metrics in physics literature (see, [2]). They also play crucial roles in the affirmative resolution of the Poincaré conjecture. In a recent study, Pigola et al. [7] introduced a natural extension of the concept of gradient Ricci solitons. They replaced the constant $\lambda$ in Ricci soliton definition with a smooth function $\lambda \in C^\infty(M)$, called soliton function, then they said that manifold is a Ricci almost soliton. They also provided existence and rigidity results and investigate some topological properties. In [8], the authors showed that if a compact Riemannian manifold admits a gradient $\eta$-Einstein soliton such that the gradient Einstein potential is a non-trivial conformal vector field, then the manifold is isometric to the Euclidean sphere. An other concept $h$-almost Ricci soliton which extends naturally the almost Ricci soliton was investigated by Gomes et al. [4]. They showed that a compact nontrivial $h$-almost Ricci soliton of dimension no less than three with $h$ having defined signal and constant scalar curvature is isometric to a standard sphere with the potential function well determined. In [3], Gahremani-Gol was investigated some equations of structure for $h$-almost Ricci soliton which are a natural generalization for almost Ricci solitons. As a result they obtained that a compact nontrivial $h$-almost Ricci soliton is isometric to a Euclidean sphere with some conditions using the Hodge–de Rham decomposition theorem. Moreover, they acquired an integral formula for the compact $h$-almost Ricci solitons. Later, some results were obtained for the solitons of the Ricci-Bourguignon flow by Dwivedi [1], generalizing corresponding results for Ricci solitons. Taking motivation from Ricci almost solitons, the author then introduced the notion of Ricci-Bourguignon almost solitons and proved some results about them which generalize previous results for Ricci almost solitons. He also derived integral formulas for compact gradient Ricci-Bourguignon solitons.
solitons and compact gradient Ricci-Bourguignon almost solitons. Finally, using the integral formula he showed that a compact gradient Ricci-Bourguignon almost soliton is isometric to an Euclidean sphere. Considering the Ricci-Bourguignon almost solitons together with $h$ almost solitons, we present a new generalized Ricci almost soliton and investigate some aspects of this generalized Ricci almost solitons in a complete Riemannian manifold. First, we prove that the compact gradient generalized Ricci almost soliton is isometric to the Euclidean sphere by showing that the scalar curvature becomes constant. Second, we show that in a compact generalized gradient Ricci almost soliton satisfying an integral condition.

**Keywords** Generalized Ricci almost soliton · Riemannian manifold · Standard sphere

**References**


ON REDUCTION OF A (2+1)-DIMENSIONAL NONLINEAR SCHröDINGER EQUATION VIA CONSERVATION LAWS

Yeşim SAĞLAM ÖZKAN

Bursa Uludag University, Art and Science Faculty Department of Mathematics, Bursa, Turkey

Corresponding Author’s E-mail: ysaglam@uludag.edu.tr

ABSTRACT

In this work we consider a (2+1)-dimensional nonlinear Schrödinger equation which is one of the important models in branches of plasma physics, nonlinear optics, fluid dynamics, etc. Considering the Lie point symmetries and conservation laws obtained with the help of methods available in the literature, the association between symmetries and conservation laws leads to a reduction in the equation. This theory reduces both the order and the number of independent variables involved in underlying equation and quite useful to obtain new solutions.

Keywords Conservation laws · Lie symmetries · Schrödinger equation

References

RINGS WHOSE G-SEMIARTINIAN MODULES HAVE MAXIMAL OR MINIMAL SUBPROJECTIVITY DOMAIN

Yılmaz DURĞUN 1, Ayşe ÇOBANKAYA 2

1,2 Department of Mathematics, Cukurova University, Turkey
Corresponding Author’s E-mail: acaylak@cu.edu.tr

ABSTRACT

G-semiartinian modules were studied and introduced in [1]. A module $M$ is called g-semiartinian if every non-zero homomorphic image of $M$ has a singular simple submodule. In this study, we introduce the modules in which g-semiartinian modules are subprojective. By Holston and et al. in [4], the subprojectivity domain were introduced and determine how far a non-projective module is from being projective. Following [4], given modules $X_1$ and $X_2$, $X_1$ is $X_2$-subprojective if for each epimorphism $\alpha : P \to X_2$ and each morphism $h : X_1 \to X_2$, there exists a morphism $f : X_1 \to P$ with $\alpha f = h$. The necessary and sufficient conditions for a module $X_1$ to be $X_2$-subprojective are given in [4, 2]. For any module $X$, $\mathcal{B}_X^{-1}(X) = \{L \in \text{Mod} - R \mid X \text{ is } L\text{-subprojective}\}$ is called the subprojectivity domain of $X$. Clearly, $X$ is projective if and only if $\mathcal{B}_X^{-1}(X) = \text{Mod} - R$. In other words, $X$ is projective if its subprojectivity domain is as large as it can be. A g-semiartinian module whose domain of subprojectivity as small as possible is called gsap-indigent. We investigated the structure of rings whose (simple, coatomic) g-semiartinian right modules are gsap-indigent or projective. Furthermore, over right PS rings, necessary and sufficient condition to be gsap-indigent module was determined. Finally, inspired by similar ideas and problems in [4, 3], gsap-indigent modules are studied. Over right PS ring, we prove that a g-semiartinian right module $Y$ is gsap-indigent if and only if $\text{Hom}(Y, S) \neq 0$ for every singular simple right $R$-module $S$. To keep in line with [5] and [3], we say $R$ has no subprojective (simple) gsa-middle class if every (simple) g-semiartinian module is either gsap-indigent or projective. It is proven that (1) If $R$ has no subprojective simple gsa-middle class, then $R$ is either right PS or right Kasch; (2) If $R$ is not right Kasch, then $R$ has no subprojective simple gsa-middle class if and only if all non-projective coatomic g-semiartinian module is gsap-indigent if and only if all non-projective modules of finite length are gsap-indigent. Also, it is shown that if $R$ is a commutative GD-extending ring, then $R$ has no subprojective simple gsa-middle class if and only if
there is a ring direct sum $R \cong S \times T$, where $S$ is semisimple Artinian ring and $T$ is an indecomposable ring which is either a local QF-ring or a local nonsingular ring.

**Keywords** Subprojectivity domain · PS ring · Kasch ring

**Acknowledgement:** This work was supported by the Scientific and Technological Research Council of Turkey (TUBITAK) (Project number: 119F176).

**References**


ON THE ★-CONGRUENCE SYLVESTER EQUATION

Yuki SATAKE¹, Tomohiro SOGABE¹, Tomoya KEMMOCHI¹, Shao-Liang ZHANG¹

¹Graduate School of Engineering, Nagoya University, Furo-cho, Chikusa-ku, Nagoya 464-8603, Japan
Corresponding Author’s E-mail: y-satake@na.nuaap.nagoya-u.ac.jp

ABSTRACT

We consider the following matrix equation

$$AX + X^*B = C,$$

where $A \in \mathbb{C}^{m \times n}$, $B \in \mathbb{C}^{n \times m}$, and $C \in \mathbb{C}^{m \times m}$ are given, and $X \in \mathbb{C}^{n \times m}$ is to be determined. The operator $(\cdot)^*$ denotes the transpose $(\cdot)^T$ or the conjugate transpose $(\cdot)^H$ of a matrix, and then equation (1) is called the ★-congruence Sylvester equation. The ★-congruence Sylvester equation appears in palindromic eigenvalue problems arising from some realistic applications such as the vibration analysis of fast trains, see, e.g., [1].

In recent paper [2], when $\star = T$ and the given matrices are square ($m = n$), it was shown that the ★-congruence Sylvester equation is mathematically equivalent to the Lyapunov equation under certain conditions. The Lyapunov equation is widely known in control theory and has been much studied. Therefore, the study [2] indicates that it can be possible to utilize the rich results on the Lyapunov equation for the ★-congruence Sylvester equation (1). Indeed, it was shown that the direct method for the Lyapunov equation can be used to obtain a numerical solution of (1) with computational cost lower than that of the conventional method [4] for (1). One of the important approaches in the previous study is to use an equivalent linear system that is obtained by vectorization. Oozawa et al. succeeded in reducing (1) to the Lyapunov equation by applying an appropriate linear operator to the linear system and returning it into a matrix [2].

The theoretical result was extended to the case where $A$ and $B$ are rectangular ($m \neq n$) [3]. In this case, it was shown that (1) is equivalent to the generalized Sylvester equation. However, when $\star = H$, the same transformation cannot be applied to (1) because (1) is a nonlinear equation.

In this talk, we consider a linearization of (1) with $\star = H$, and construct an appropriate operator to obtain an equivalent linear matrix equation. We will show that (1) is equivalent to the generalized Sylvester equation under certain conditions, and introduce numerical solvers utilizing our results.
Keywords  ⋆-Congruence Sylvester equation · Matrix equation · Linear operator

References

Participant List

ARACELI QUEIRUGA-DIOS, (SPAIN), Invited Speaker
AYMAN BADAWI, (UAE), Invited Speaker
CARLOS MARTINS DA FONSECA, (KUWAIT), Invited Speaker
ABDISALAM HASSAN MUSE, (SOMALIA), Listener
AGUSTIN MARTIN, (SPAIN), Listener
ALDINA CORREIA, (PORTUGAL), Oral Presentation
ALEJANDRO MEDINA, (SPAIN), Oral Presentation
ALICE ANGELESCU, (ROMANIA), Listener
ANA BORGES, (PORTUGAL), Oral Presentation
ASCENSIÓN HERNÁNDEZ, (SPAIN), Listener
ASUKA OHASHI, (JAPAN), Oral Presentation
AYŞE ÇOBANKAYA, (TURKEY), Oral Presentation
B. ALİ İBRAHİMOĞLU, (TURKEY), Oral Presentation
BAHAR KARAMAN, (TURKEY), Oral Presentation
BAHAR KULOĞLU, (TURKEY), Oral Presentation
BASＲİ ÇALIŞKAN, (TURKEY), Oral Presentation
BUSE İＮＧＥＮÇ, (TURKEY), Oral Presentation
CRISTINA CARIDADE, (PORTUGAL), Oral Presentation
CUMALI YOLDAŞ, (TURKEY), Listener
DANIＥLA РИＣТАРИКОВА, (SLOVAKIA), Oral Presentation
DENIZ ALTUN, (TURKEY), Oral Presentation
DEOLINDA MARＩA DIＡS RАＳТЕＩＲО, (PORTUGAL), Oral Presentation
DIN PRÂTHUMWAN, (THAILAND), Oral Presentation
DURDУ HAKАＮ UTКУ, (TURKEY), Oral Presentation
DУRSУN TАＳＣІ, (TURKEY), Listener
DＵＹＧＵ ÇＡＧΛＡＲ, (TURKEY), Oral Presentation
ＥL AMＩR DИEFFАЛ, (ALGERIA), Oral Presentation
ELIANA COSTA E SILVA, (PORTUGAL), Oral Presentation
Emel Karaca, (Turkey), Oral Presentation
Emrah Karaman, (Turkey), Oral Presentation
Emrah Yildirim, (Turkey), Oral Presentation
Engin Özkân, (Turkey), Listener
Erdal Ünlüyol, (Turkey), Oral Presentation
Fatih Kasimoglu, (Turkey), Oral Presentation
Fatma Karakuş, (Turkey), Oral Presentation
Fatma Dişdem Köparal, (Turkey), Oral Presentation
Figen Kangalgil, (Turkey), Oral Presentation
Gerardo Rodriguez Sanchez, (Spain), Listener
Gökhan Mutlu, (Turkey), Oral Presentation
Gül Uğur Kaymanlı, (Turkey), Oral Presentation
Hakan Aykut, (Turkey), Oral Presentation
Inthira Chaïya, (Thailand), Oral Presentation
Ion Mierlus Mazilu, (Romania), Oral Presentation
İnan Ünal, (Turkey), Oral Presentation
İpek Ebru Karaçay, (Turkey), Oral Presentation
İsmail Aslan, (Turkey), Oral Presentation
J.M. López Belinchón, (Spain), Oral Presentation
Jana Gabková, (Slovakia), Listener
Jesus Martin Vaquero, (Spain), Listener
Jose Maria Chamoso, (Spain), Listener
Kamran Zakaria, (Pakistan), Oral Presentation
Lucian Nita, (Romania), Listener
Luis Hernández Encinas, (Slovakia), Listener
Mahmut Modanlı, (Turkey), Oral Presentation
Maria Aydin, (Turkey), Oral Presentation
Maria Emília Bigotte de Almeida, (Portugal), Oral Presentation
Maria Jesus Santos, (Spain), Listener
MARÍA JOSÉ CÁCERES, (SPAIN), LISTENER
MARIE DEMLOVA, (CZECH REPUBLIC), LISTENER
MD. ALAL HOSEN HOSEN, (BANGLADESH), ORAL PRESENTATION
MELEK SOFYALIOĞLU, (TURKEY), ORAL PRESENTATION
MELEK YAĞCI, (TURKEY), ORAL PRESENTATION
MELTEM EKİZ, (TURKEY), ORAL PRESENTATION
MERT AKIN İNSEL, (TURKEY), ORAL PRESENTATION
MERVE ULUDAĞ, (TURKEY), ORAL PRESENTATION
MERVE TaŞTAN, (TURKEY), ORAL PRESENTATION
MICHAEL CARR, (IRELAND), LISTENER
MIGUEL ANGEL LOPEZ GUERRERO, (SPAIN), LISTENER
MILICA ANDELIC, (KUWAIT), ORAL PRESENTATION
MUAMMER AYATA, (TURKEY), ORAL PRESENTATION
MUKADDES ÖKtener TURACI, (TURKEY), ORAL PRESENTATION
MUSA ÇAKMAK, (TURKEY), ORAL PRESENTATION
MUSTAFA AGGUL, (TURKEY), ORAL PRESENTATION
MUSTAFA AŞCı, (TURKEY), ORAL PRESENTATION
MUSTAFA ÇALIŞKAN, (TURKEY), LISTENER
MUSTAFA ÖZKAN, (TURKEY), LISTENER
MUSTAFA ALTıN, (TURKEY), ORAL PRESENTATION
MUSTAFA AŞLANTAŞ, (TURKEY), ORAL PRESENTATION
MÜCAHİT AKBİYIK, (TURKEY), LISTENER
NURTEN GÜRSES, (TURKEY), LISTENER
ONUR ŞAHİN, (TURKEY), ORAL PRESENTATION
PAUL ROBINSON, (IRELAND), LISTENER
PETER LETAVAJ, (SLOVAKIA), ORAL PRESENTATION
RABIA ÇAKAN AKPİNAR, (TURKEY), ORAL PRESENTATION
RAMAZAN ÖZARSLAN, (TURKEY), ORAL PRESENTATION
RAMAZAN SARI, (TURKEY), ORAL PRESENTATION
RAUL ALCARAZ, (SPAIN), LISTENER
REMZI AKTAY, (TURKEY), ORAL PRESENTATION
SABA INAM, (PAKISTAN), LISTENER
SABRINA FRANCESCA PELLEGRINO, (ITALY), ORAL PRESENTATION
SENDER YAMAÇ AKBIYIK, (TURKEY), LISTENER
SEHER SULTAN YEŞILKAYA, (TURKEY), ORAL PRESENTATION
SELAMI BAYEG, (TURKEY), ORAL PRESENTATION
SELÇUK ÖZCAN, (TURKEY), LISTENER
SELIHAN KIRLAK, (TURKEY), ORAL PRESENTATION
SEVAL İŞIK, (TURKEY), ORAL PRESENTATION
SINAN SERKAN BILGICI, (TURKEY), ORAL PRESENTATION
SNEZHANA GOCHEVA-ILIeva, (BULGARIA), ORAL PRESENTATION
STEFANIA CONSTANTINESCU, (ROMANIA), LISTENER
SÜLEYMAN AYDINYÜZ, (TURKEY), ORAL PRESENTATION
SÜMEYYE EKER, (TURKEY), ORAL PRESENTATION
TUFAñ TURACI, (TURKEY), ORAL PRESENTATION
TUĞBA MERT, (TURKEY), ORAL PRESENTATION
VÍCTOR GAYOSO MARTíNEZ, (SPAIN), ORAL PRESENTATION
VILDAN ÖZTÜRK, (TURKEY), LISTENER
YASEMIN SOYLU, (TURKEY), ORAL PRESENTATION
YEŞİM SAGLAM ÖZKAN, (TURKEY), ORAL PRESENTATION
YUKI SATAKE, (JAPAN), ORAL PRESENTATION